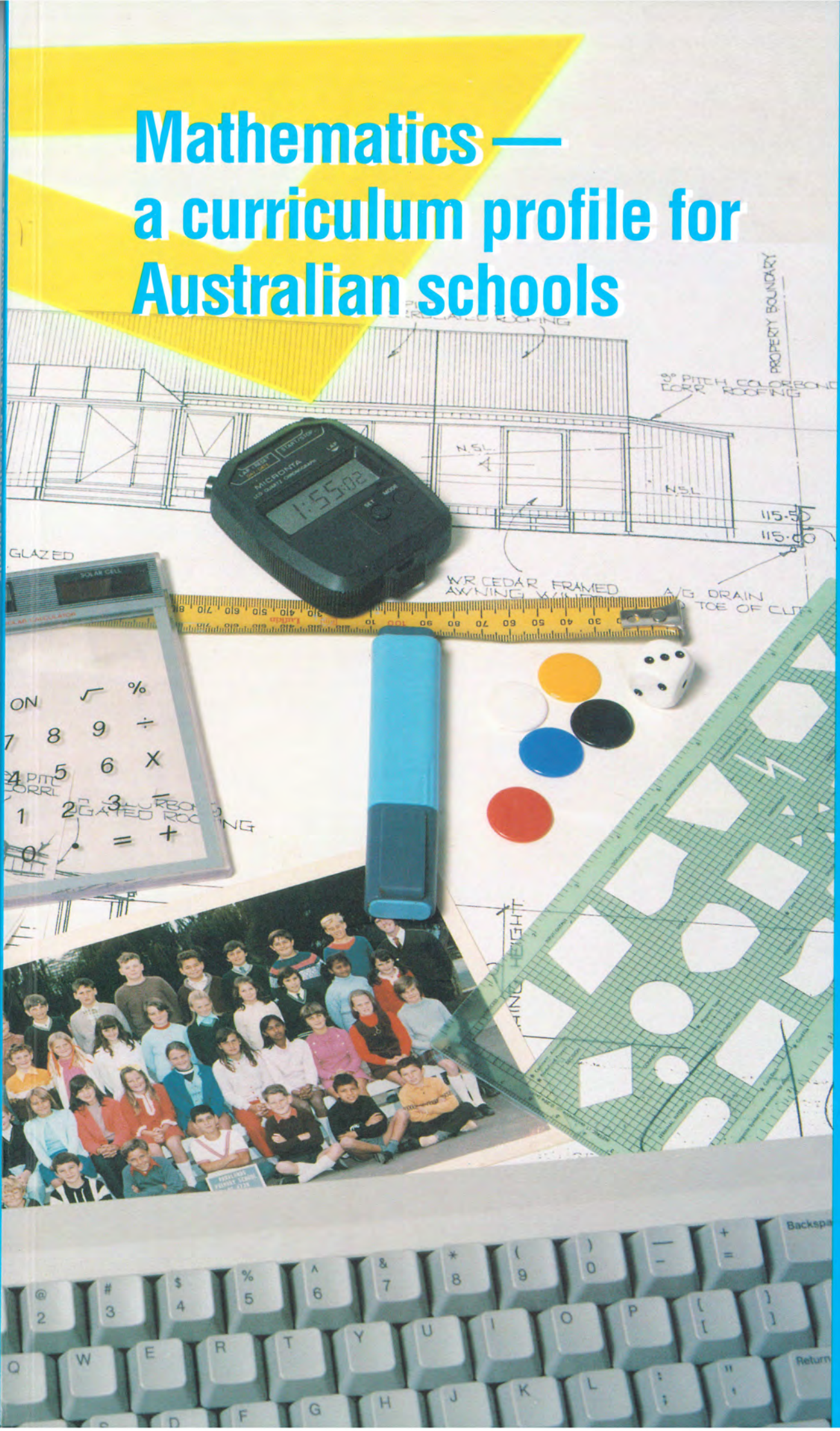
Mathematics — a curriculum profile for Australian schools



Mathematics

Mathematics — a curriculum profile for Australian schools

A joint project of the States, Territories and the Commonwealth of Australia initiated by the Australian Education Council

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Foreword

This volume is one of a series of documents which together represent the most significant collaborative curriculum development project in the history of Australian education.

National collaboration has produced sixteen documents: a statement and a profile in each of eight areas of learning — English, mathematics, science, technology, languages other than English, health and physical education, studies of society and environment, and the arts. The sixteen documents are published in seventeen volumes, since the mathematics profile is published in two volumes.

In April 1989 the State, Territory and Commonwealth Ministers of Education endorsed ten common and agreed national goals for schooling in Australia. Over the following years, work proceeded on the development of statements and profiles. This work was undertaken at the direction of the Australian Education Council (AEC), the national council of Ministers of Education.

The statements in the eight areas of learning provide a framework for curriculum development by education systems and schools. They are divided into strands which reflect the major elements of learning in each area. Further, they are structured in four bands, roughly corresponding to the stages of schooling: lower primary, upper primary, junior secondary and post-compulsory. The statements do not provide a syllabus. Rather, they provide a foundation for courses which will meet students' needs and reflect advances in our knowledge — both of the learning area to which the statement is related and of how students learn. The statements encourage innovation and experimentation so that students have a positive experience of each learning area.

The profiles are designed to assist in the improvement of teaching and learning and to provide a common language for reporting student achievement. They are divided into strands for each learning area. Within each strand, eight achievement levels have been developed. Overall, the eight levels reflect the full range of student achievement during the compulsory years of schooling (Years 1–10). The Australian Council for Educational Research (ACER) has validated the levels. The profiles have also been subject to intensive trialling in Australian schools.

The project was managed by the AEC Curriculum and Assessment Committee (CURASS), chaired most recently by the New South Wales Director-General of School Education, Dr Ken Boston. CURASS included representation from the Commonwealth, States and Territories, New Zealand, Catholic and independent schools, parents, teachers, the AEC Secretariat, ACER and Curriculum Corporation. CURASS was supported by a secretariat with representation from all States and Territories and the Commonwealth.

Project teams were established to undertake the writing, while specialist staff from States and Territories and the Commonwealth assisted with development. In each learning area consultants were appointed with responsibility for ensuring that gender equity and Aboriginal and Torres Strait Islander perspectives were reflected in the documents. Throughout the writing process, nationwide consultations occurred with groups such as parents, teachers (from both government and non-government sectors), teacher educators, professional associations, subject and discipline specialists, curriculum developers, community groups, employers and unions.

At its meeting in July 1993, the AEC agreed that the publication of statements and profiles shall be the prerogative of each State and Territory. The Board of Curriculum Corporation in accordance with the wish of member systems is publishing, disseminating and marketing these materials developed through national collaborative processes.

David Francis
Executive Director

Contents

Introduction

The States, Territories and the Commonwealth have, since 1989, worked together on a major national educational initiative to produce statements and profiles in eight broad areas of learning:

The arts	English
Health and physical education	Languages other than English
Mathematics	Science
Studies of society and environment	Technology.

The Australian Education Council (AEC), made up of the education ministers of the States, Territories and Commonwealth, commissioned the work.

Statements provide a framework for curriculum development in each area of learning. They define the area, outline its essential elements, show what is distinctive about it and describe a sequence for developing knowledge and skills.

Statements provide an account of the strands and bands of each learning area. Strands are groupings of understandings of a learning area's content, processes and concepts. Bands are the broad stages in a sequence for developing knowledge, understandings and skills in a learning area. Each statement has four bands. Generally, Bands A and B will be covered in primary schooling, C in secondary school to year 10, and D in the post-compulsory years.

Profiles describe the progression of learning typically achieved by students during the compulsory years of schooling (Years 1–10) in each of the areas of learning. Their purpose is twofold: to help teaching and learning and to provide a framework for reporting student achievement. Profiles are divided into strands, usually the same as those in the statement, and into eight levels of achievement.

Profiles and statements are linked. The profiles show the typical progression in achieving learning outcomes, while statements are a framework of what might be taught to achieve these outcomes.

This profile is the result of national collaboration in the mathematics learning area. It drew upon:

- A curriculum map of mathematics, completed in 1989. Like other curriculum maps developed as part of the national collaborative curriculum projects, this described the curriculum in each State and Territory, identifying similarities and differences.
- The mathematics statement.
- Responses from three periods of formal consultation, national trials and two validations carried out by the Australian Council of Educational Research.

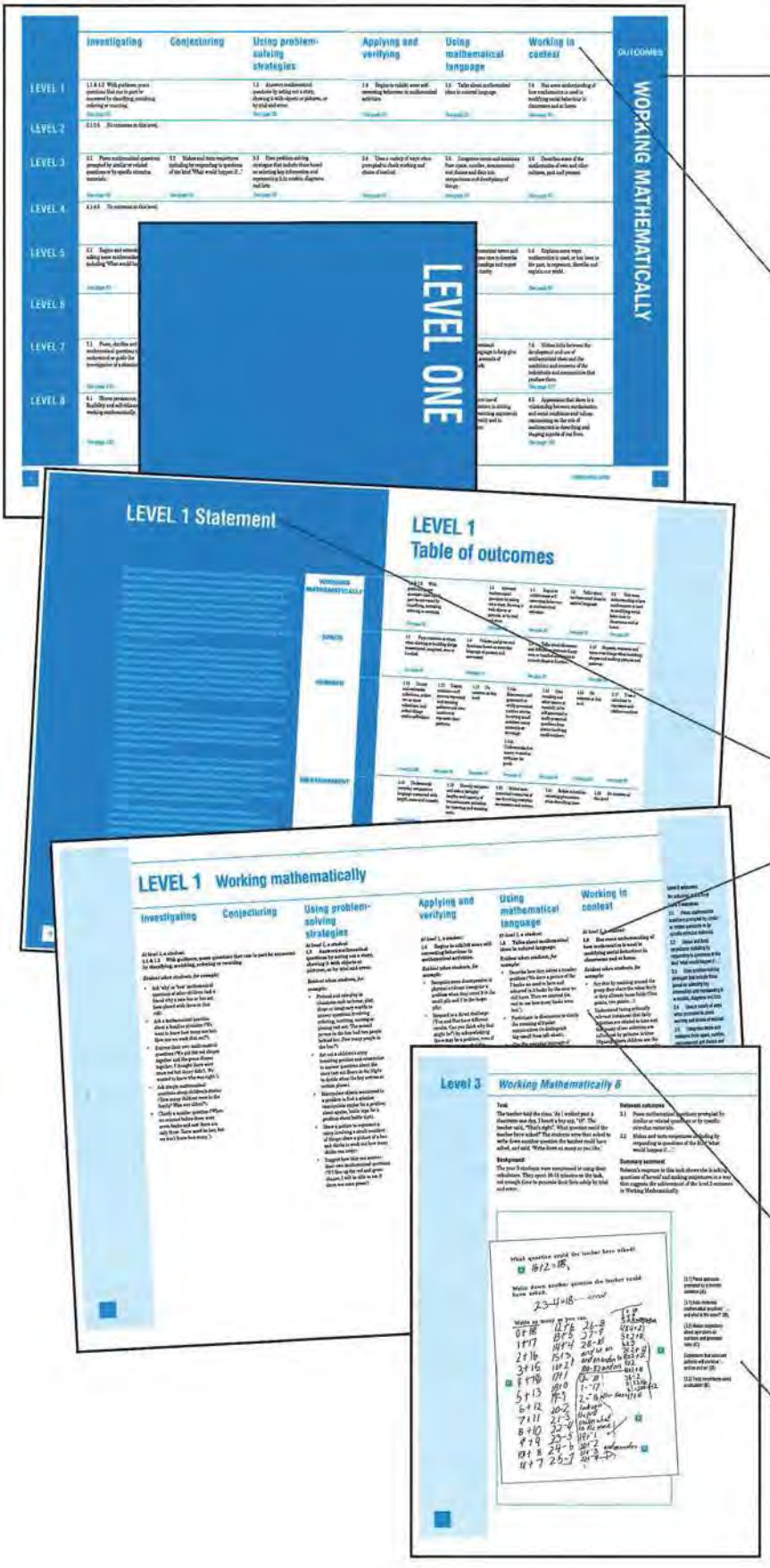
Details about the history of national collaboration on curriculum can be found in the Appendix.

To assist in meeting outcomes and ensure the participation of all students, please note:

- Students with disabilities may need specialised equipment such as computers with appropriate software and peripherals (for example, large print, voice output, calculators with voice output or large display), braillers, raised-line drawing kits, interlocking blocks, templates, tactile rulers and tape measures, closed-circuit TV, or other types of magnification aids, such as a hand-held magnifier, telescope for distance work.

- Physical help may be needed when students are required to 'build', 'design and construct', 'move up', 'measure', 'sketch', 'draw', 'classify', 'match', 'plan and make', 'interpret and make', 'arrange', 'select and fit together', 'sort', 'cut'.
- The terms 'talk about', 'say', 'oral' are understood to include all forms of communication, including signed communication (Auslan, Signed English) or the use of communication aids (boards, Compic, Canon Communicators).
- The term 'look at' can be defined as attending to, particularly for vision and hearing impaired students.
- For students with marked vision impairment:
    - The concept 'visualise' may be expressed physically.
    - Materials and books may need to be presented in alternate formats such as large print, brailled, taped, tactile drawings, diagrams, raised graphs.
    - Concepts such as 'bird's eye view', translation/rotation/reflection, will need to be taught using 3D examples.

Elements of the profile



Strand display

Strands are the major organisers of a learning area. They can be groupings of content, process and/or conceptual understanding.

This profile is organised around six strands, five of which are based on important mathematical concepts and content: Space; Number; Measurement; Algebra; Chance and data. The sixth strand, Working mathematically, deals with the processes used in all studies in the area.

Strand Organisers are organisers of content, process and/or conceptual understanding within a strand. Space, for example, has a double line of outcomes dealing with students' use of spatial ideas, tools and techniques to interpret drawings, draw and make 2D figures and 3D objects. The process strand has a line of outcomes about using problem-solving strategies. The strand organiser is indicated by the number after the decimal point in the number sequence before each outcome.

Level display

Levels indicate progression in student learning. There are eight levels covering the compulsory years of schooling (Years 1–10). The level is indicated by the number before the decimal point at the beginning of each outcome.

Level statements are general descriptions of student performance at each of the eight levels within the profile.

Outcomes describe in progressive order the various skills and knowledge that students typically acquire as they become more proficient in an area. They are the building blocks of the profile.

Not all lines of outcomes are complete. Working mathematically has outcomes at level one, three, five, seven and eight. It is hoped that, as the profile is reviewed, outcomes in the other levels will be identified.

Almost all strands have fewer outcomes at level eight than any other level. This is because level eight is intended to represent higher achievement than level seven but not extra content. Students at level eight show more sophisticated conceptualisation and skill in dealing with the same content than students at level seven.

Pointers are indicators or signals of the achievement of an outcome. Unlike outcomes, pointers are only examples. Other pointers not mentioned could also indicate achievement of the outcome. Bracketed sections are examples that further develop pointers. The brackets indicate a sample from a larger set of items.

Annotated work samples show student work which demonstrates the achievement of one or more outcomes at a level. The samples are annotated to show the reasons for this judgement.

The mathematics work samples are provided in a separate volume to the profile.

	Investigating	Conjecturing	Using problem-solving strategies
LEVEL 1	1.1 & 1.2 With guidance, poses questions that can in part be answered by classifying, matching, ordering or counting. See page 22		1.3 Answers mathematical questions by acting out a story, showing it with objects or pictures, or by trial and error. See page 22
LEVEL 2	2.1-2.6 No outcomes at this level.		
LEVEL 3	3.1 Poses mathematical questions prompted by similar or related questions or by specific stimulus materials. See page 52	3.2 Makes and tests conjectures including by responding to questions of the kind 'What would happen if...?'. See page 52	3.3 Uses problem-solving strategies that include those based on selecting key information and representing it in models, diagrams and lists. See page 52
LEVEL 4	4.1-4.6 No outcomes at this level.		
LEVEL 5	5.1 Begins and extends tasks by asking some mathematical questions, including 'What would happen if...?'. See page 82	5.2 Understands a conjecture as a guess with reasons and draws on mathematical knowledge to give reasons for conjectures before testing them. See page 82	5.3 Uses problem-solving strategies, including those based on selecting and organising key information and being systematic. See page 82
LEVEL 6	6.1-6.6 No outcomes at this level.		
LEVEL 7	7.1 Poses, clarifies and refines mathematical questions to help understand or guide the investigation of a situation. See page 116	7.2 Makes generalisations by abstracting common mathematical features from situations, tests with additional cases and explains why generalisations must be true. See page 116	7.3 Uses problem-solving strategies that include identifying and working on related problems or sub-problems. See page 116
LEVEL 8	8.1 Shows persistence, autonomy, flexibility and self-reliance when working mathematically. See page 132	8.2 Produces mathematical arguments to convince others of the truth of propositions, including those involving deductions from known information. See page 132	8.3 Uses problem-solving strategies that include those based on generalising from one problem situation to another and rethinking problem conditions and constraints. See page 132

Applying and verifying	Using mathematical language	Working in context
1.4 Begins to exhibit some self-correcting behaviour in mathematical activities. See page 23	1.5 Talks about mathematical ideas in natural language. See page 23	1.6 Has some understanding of how mathematics is used in modifying social behaviour in classrooms and at home. See page 23
3.4 Uses a variety of ways when prompted to check working and choice of method. See page 53	3.5 Integrates terms and notations from space, number, measurement and chance and data into comparisons and descriptions of things. See page 53	3.6 Describes some of the mathematics of own and other cultures, past and present. See page 53
5.4 Checks that answers fit specifications and make sense in the original situation. See page 83	5.5 Uses mathematical terms and notations with some care to describe objects and relationships and report conclusions with clarity. See page 83	5.6 Explains some ways mathematics is used, or has been used in the past, to represent, describe and explain our world. See page 83
7.4 Applies standard methods or models, comparing and choosing between alternatives, including by considering assumptions needed and results obtained. See page 117	7.5 Uses conventional mathematical language to help give clear and logical accounts of mathematical work. See page 117	7.6 Makes links between the development and use of mathematical ideas and the conditions and concerns of the individuals and communities that produce them. See page 117
8.4 Formulates models by making useful and simple assumptions, collecting data needed and representing the relevant relationships in mathematical terms. See page 133	8.5 Makes fluent use of mathematical notation in solving problems and presenting arguments succinctly, coherently and in conventional forms. See page 133	8.6 Appreciates that there is a relationship between mathematics and social conditions and values, commenting on the role of mathematics in describing and shaping aspects of our lives. See page 133

	Using spatial ideas, tools and techniques to interpret, draw and make	Visualising, analysing and representing arrangements and locations
LEVEL 1	1.7 Pays attention to shape when drawing or building things remembered, imagined, seen or handled. See page 24	1.8 Follows and gives oral directions based on everyday language of position and movement. See page 24
LEVEL 2	2.7a Fulfils simple spatial criteria when making things from verbal or visual descriptions. 2.7b Pays attention to the shape and placement of component parts when interpreting and making drawings. See page 38	2.8 Follows and gives descriptions of locations and paths and pays attention to order in reading and making informal maps. See page 38
LEVEL 3	3.7a Pays attention to the shape and placement of parts when matching, making and copying things, including matching nets with 3D shapes. 3.7b Matches actual models and conventional drawings of them and attends to what can be seen when making drawings. See page 54	3.8 Visualises, follows and gives descriptions of locations and paths and attends to order and proximity in reading and making maps. See page 54
LEVEL 4	4.7a Shows care in the shape, size and placement of parts when they match, make and copy things, including making nets of 3D shapes by drawing around their faces. 4.7b Interprets and makes drawings of 3D shapes, using basic conventions for representing 3D space in 2D. See page 68	4.8 Visualises, follows and describes locations and paths and reads and makes maps and plans, using distance, direction, coordinates and scales. See page 68
LEVEL 5	5.7a Makes accurate versions of simple mathematical figures and objects, including making own nets. 5.7b Interprets and makes drawings of 3D shapes accurately, using conventions for representing 3D space in 2D with consistency. See page 84	5.8 Uses network diagrams to represent the order of, and paths between, locations and events. See page 84
LEVEL 6	6.7 Uses geometric techniques and tools to interpret and meet specifications requiring the accurate construction and placement of figures and objects. See page 100	6.8 Visualises, sketches and describes paths and regions that satisfy conditions given in everyday language. See page 100
LEVEL 7	7.7 Draws on properties of shapes and transformations to plan how to meet specifications requiring the accurate construction or placement of figures and objects. See page 118	7.8 Visualises, constructs and describes paths and regions using conventional geometric language, including that based on coordinates. See page 118
LEVEL 8	8.7–8.10 Recognises the systematic nature of a geometry and draws flexibly upon, and sees connections between, results about shapes, transformations and locations. See page 134	

Visualising, analysing and representing shapes	Visualising, analysing and representing movements and transformations
1.9 Talks about likenesses and differences between things seen or handled and begins to connect shape to function. See page 25	1.10 Repeats, reorients and turns over things when matching shapes and making pictures and patterns. See page 25
2.9 Interprets common spatial language and, when prompted, uses it to describe the shape of things. See page 39	2.10 Generates patterns and follows rules based on the simple repetition and movement of things. See page 39
3.9 Interprets common spatial language and uses it to describe and compare features of things. See page 55	3.10 Recognises and uses repetitions and movements of the same shape embedded within arrangements and patterns. See page 55
4.9 Selects, describes and compares figures and objects on the basis of spatial features, using conventional geometric criteria. See page 69	4.10 Recognises and uses rotations, reflections and translations to relate the features of arrangements and patterns. See page 69
5.9 Analyses, describes and uses distinguishing features of common classes of mathematical figures and objects, recognising parallels, perpendiculars and congruence. See page 85	5.10 Visualises, moves and sketches shapes to show the effect of translations, reflections, rotations and enlargements (using grids). See page 85
6.9 Analyses, describes and uses relationships in and between classes of figures, including parallel, perpendicular and intersecting lines, triangles and rectangles. See page 101	6.10 Visualises, produces and accurately describes translations, reflections, rotations and enlargements. See page 101
7.9 Investigates and uses relationships in and between classes of figures, including quadrilaterals and circles. See page 119	7.10 Analyses translations, reflections, rotations and enlargements and relates their properties to similarity and congruence. See page 119

	Count and order	Number patterns	Equations
LEVEL 1	1.11 Counts and estimates collections, orders two or more collections, and orders things within collections. See page 26	1.12 Copies, continues and invents repeating and counting patterns and uses numbers to represent their patterns. See page 26	1.13 No outcome at this level.
LEVEL 2	2.11 Counts, orders, estimates and describes with whole numbers within everyday experience and with fractions expressed in words. See page 40	2.12 Identifies, continues and invents whole number patterns, including where successive terms in a sequence can be linked by addition or subtraction of a constant amount. See page 40	2.13 Uses understanding of whole numbers and addition and subtraction relationships to construct and complete simple statements of equality. See page 41
LEVEL 3	3.11 Counts, orders, estimates and describes with whole numbers, common fractions and decimal fractions for money or measurements (two places). See page 56	3.12 Identifies, continues and invents whole number patterns involving the four operations, including where successive terms in a sequence can be linked by an addition or subtraction strategy. See page 56	3.13 Uses understanding of whole and fractional numbers, relationships and operations to construct and complete simple statements of equality. See page 57
LEVEL 4	4.11 Counts, orders, estimates and describes with common and decimal fractions. See page 70	4.12 Identifies, continues and invents whole and fractional number patterns, including those where successive terms in a sequence can be linked by a multiplication or division strategy. See page 70	4.13 Uses understanding of numbers and number relationships to construct and complete statements of equality, including those where more than one solution exists. See page 71
LEVEL 5	5.11 Interprets and uses whole powers and square roots and straightforward ratios and percentages. See page 86	5.12 See outcome 5.28 See page 94	5.13 See outcome 5.30 See page 95
LEVEL 6	6.11 Interprets and uses whole powers and roots, scientific notation, ratios, percentages and negative numbers. See page 102	6.12 See outcome 6.28 See page 110	6.13 See outcome 6.30 See page 111
LEVEL 7	7.11 Selects a suitable form of number representation, explains choice and moves freely between representations. See page 120	7.12 See outcome 7.28 See page 126	7.13 See outcome 7.30 See page 127
LEVEL 8	8.11–8.17 Searches for and uses representations for number and operations that will assist the solution of problems by highlighting patterns in numbers or by reducing complexity and computational load. See page 135		

Applying numbers	Mental computation	Written computation	Calculators
1.14a Represents self-generated or orally presented number stories involving small numbers using materials or drawings. 1.14b Understands that money is used in exchange for goods. See page 27	1.15 Uses counting and other means to mentally solve self-generated or orally presented questions from stories involving small numbers. See page 28	1.16 No outcome at this level.	1.17 Uses a calculator to represent and explore numbers. See page 29
2.14a Represents problems involving the four basic operations on whole numbers in a variety of ways, using stories, materials, pictures and number sentences. 2.14b Compares prices with money available, calculates the exact and approximate cost of several items and makes change. See page 41	2.15 Estimates and calculates mentally, including adding and subtracting numbers to 10 and making extensions based on place value. See page 42	2.16 Uses a variety of strategies, including regrouping, to assist in adding and subtracting whole numbers when unable to complete mentally. See page 42	2.17 Uses a calculator to represent and explore numbers, place value and operations. See page 43
3.14 Makes a suitable choice of operation for situations involving whole numbers, amounts of money and familiar measurements. See page 57	3.15 Estimates and calculates mentally, including adding (sum to 100) and subtracting two-digit numbers and multiplying numbers to 10. See page 58	3.16 Uses understood written methods to add and subtract any whole numbers and amounts of money and to multiply and divide whole numbers by whole numbers to 10. See page 58	3.17 Uses a calculator for operating on whole numbers, amounts of money and measurements. See page 59
4.14 Makes a suitable choice of operation involving whole and fractional numbers (whole number multipliers and divisors), including those where more than one operation is needed. See page 71	4.15 Estimates and calculates mentally, including adding and subtracting most two-digit numbers and multiplying and dividing multiples of 10 by one-digit numbers. See page 72	4.16 Uses understood written methods to add, subtract, multiply and divide whole numbers, money and measures (two places, whole number multipliers and divisors to 10). See page 72	4.17 Uses a calculator efficiently for operating on decimal numbers, including where more than one operation is needed, and interprets displays for division. See page 73
5.14 Chooses and sequences several operations, covering situations involving decimal multipliers and divisors and division of smaller by larger numbers. See page 87	5.15 Estimates and calculates mentally with whole numbers, money and simple fractions, including multiplying and dividing some two-digit numbers by one-digit numbers. See page 88	5.16 Uses understood written methods to add, subtract, multiply and divide whole numbers and common and decimal fractions (whole number multipliers and divisors). See page 88	5.17 Makes efficient use of a calculator for common and decimal fractions and percentages and takes into account orders of operations. See page 89
6.14 Uses a ratio or familiar rate to describe the relationship between two directly proportional quantities and to calculate one quantity from another. See page 103	6.15 Estimates and calculates mentally with whole and fractional numbers, including finding frequently used fractions and percentages of amounts. See page 104	6.16 Uses understood written methods to calculate with decimal and common fractions and integer powers. See page 104	6.17 Makes efficient use of a scientific calculator, including for powers and roots and using scientific notation. See page 105
7.14 Understands the nature of a rate, and chooses, calculates and compares with ratios and rates, including situations involving direct and indirect proportion. See page 120	7.15–7.17 Undertakes efficient computations on positive and negative numbers of any size, including rearranging formulae and quoting results to a suitable level of accuracy. See page 121		

	Choosing units	Measuring
LEVEL 1	1.18 Understands everyday comparative language associated with length, mass and capacity. See page 30	1.19 Directly compares and orders 'straight' lengths and capacities of two containers, including by repeating and counting units. See page 30
LEVEL 2	2.18 Chooses the appropriate physical attribute when comparing and measuring things and units which relate to that attribute. See page 44	2.19 Directly and indirectly compares lengths and capacities, and measures and makes lengths and distances by counting uniform units, including metres and centimetres. See page 44
LEVEL 3	3.18 Selects suitable and uniform things to use as units when measuring and a common unit to compare two things. See page 60	3.19 Directly and indirectly compares things, including by counting uniform units of angle, area, capacity and mass and measuring length to the nearest marked graduation. See page 60
LEVEL 4	4.18 Selects appropriate attributes and units of a sensible size for the descriptions and comparisons to be made. See page 74	4.19 Measures and makes things, using conventional units and measuring equipment for length, mass, capacity and angle and reading scales to the nearest marked graduation. See page 74
LEVEL 5	5.18 Takes purpose and practicality into account when selecting attributes, units and instruments for measuring things. See page 90	5.19 Measures and makes things, using a range of graduated scales and strategies for making measurements that are more accurate than the available equipment allows. See page 90
LEVEL 6	6.18 & 6.19 Decides what measurements are needed to complete a practical task and how to obtain them and make or collect measurements to the planned level of accuracy. See page 106	
LEVEL 7	7.18 & 7.19 No outcomes at this level.	
LEVEL 8	8.18–8.22 Selects and integrates mathematical ideas, relationships and information, directly and indirectly, to solve practical and analytic measurement problems. See page 136	

Estimating	Time	Using relationships
1.20 Makes non-numerical estimates of size involving everyday movements and actions. See page 31	1.21 Refers to familiar recurring phenomena when describing time. See page 31	1.22 No outcome at this level.
2.20 Estimates the order of things by length, area, mass and capacity and makes numerical estimates of length using a unit that can be seen or handled. See page 45	2.21 Locates and sequences events in time, beginning to read familiar clocks and calendars. See page 45	2.22 No outcome at this level.
3.20 Makes sensible numerical estimates using units that can be seen or handled and uses words such as 'between' to describe estimates. See page 61	3.21 Measures time and duration of time, reading clocks, calendars and straightforward timetables. See page 61	3.22 No outcome at this level.
4.20 Uses the known size of familiar things to help make and improve estimates, including those with centimetres, metres, kilograms and litres. See page 75	4.21 Estimates and measures time and duration of time and prepares feasible timetables. See page 75	4.22 Understands and uses relationships involving perimeters of polygons and areas of regions based on squares. See page 75
5.20 Makes sensible estimates of length, area, mass and capacity in common standard units and identifies unreasonable estimates of things. See page 91	5.21 Estimates, measures and calculates time and duration of time and uses timelines and a range of types of timetable. See page 91	5.22 Understands and uses relationships involving length, area and volume, including those for shapes based on rectangles, triangles and rectangular prisms. See page 91
6.20 Unprompted, estimates in situations in which it is sensible to do so and judges the reasonableness of estimates. See page 107	6.21 Integrates information from several sources to determine time and duration of time and to plan and synchronise events. See page 107	6.22a Understands and uses relationships involving length, area and volume of quadrilaterals and circles, prisms and pyramids. 6.22b Understands and uses similarity and Pythagoras's theorem to solve problems involving right triangles and scale drawing. See page 107
7.20 Appreciates that all measurements involve error and estimates the extent of uncertainty in direct and indirect measures. See page 123	7.21 No outcome at this level.	7.22a Selects and uses, both directly and indirectly, formulae for length, area and volume of figures and objects (including for spheres). 7.22b Understands and uses similarity relationships in and between figures and objects, including with trigonometric ratios. See page 123

	Understanding, estimating and measuring chance variation	Collecting data
LEVEL 1	1.23 Shows some recognition of the element of chance in familiar daily activities. See page 32	1.24 With guidance, poses questions about collected objects and information. See page 32
LEVEL 2	2.23 Distinguishes possible from impossible events, and describes familiar and easily understood events as more likely or less likely to happen. See page 46	2.24 Contributes questions in group activities and realises some can be answered by collecting data in the form of objects or information. See page 46
LEVEL 3	3.23 Distinguishes certain from uncertain things, describes familiar easily understood events as having equal chance of happening or being more or less likely. See page 62	3.24 Contributes to discussions to clarify what data would help answer particular questions or test predictions, and takes care in collecting data. See page 62
LEVEL 4	4.23 Places events in order from those least to most likely to happen on the basis of numerical and other information about the events. See page 76	4.24 Collaborates in deciding how data collection could help investigate situations and problems, frames helpful questions and decides what data to collect. See page 76
LEVEL 5	5.23 Interprets and makes numerical statements of probability based on lists of equally likely outcomes and using fractions and percentages. See page 92	5.24 Collaborates in planning and refining survey questions and observation methods for collecting frequency and measurement data. See page 92
LEVEL 6	6.23 Estimates probabilities and proportions based on primary or secondary data collection and assigns probabilities for one- and two-stage events by reasoning about equally likely events. See page 108	6.24 Plans experiments and surveys, collaboratively and independently, deciding how to collect data consistently and the sources and types of data. See page 108
LEVEL 7	7.23 Estimates probabilities, proportions, means and medians based on primary and secondary data collection and assigns probabilities using complementarity and independence. See page 124	7.24 Plans experiments, simulations and surveys, collaboratively and independently, considering the appropriateness and quality of observations and the suitability of samples. See page 124
LEVEL 8	8.23–8.27 Comments critically on the strengths and weaknesses of various forms of data collection, analysis and display in terms of what information can be obtained from them and what conclusions might be drawn. See page 137	

Organising data	Displaying and summarising data	Interpreting data
1.25 Participates in classifying and sequencing objects or pictures.	1.26 & 1.27 Displays objects and pictures and describes data in words and numbers.	1.28-1.30 No outcomes at this level.
See page 33	See page 33	
2.25 Contributes to deciding how to classify and sequence data, applying unambiguous and familiar criteria consistently.	2.26 Displays and summarises data based on one-to-one correspondence between data and representation.	2.27 Describes, orally and in writing, what own and classmates' displays of data show.
See page 47	See page 47	See page 47
3.25 Classifies, sequences and tabulates data to help answer particular questions and varies the classification to answer different questions.	3.26 Displays and summarises data using frequencies, measurements and many-to-one correspondence between data and representation.	3.27 Reads and describes information in simple tables, diagrams, pictographs and bar graphs.
See page 63	See page 63	See page 63
4.25 Classifies, sequences and tabulates data, with some grouping of data, and chooses methods helpful for answering particular questions.	4.26 Displays frequency and measurement data using simple scales on axes and summarises data with simple fractions, highest, lowest and middle scores, and means.	4.27 Reads and describes information in tables (including some grouping of data), diagrams, line and bar graphs, fractions and means.
See page 77	See page 77	See page 77
5.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, using class intervals and fields provided or planned with help.	5.26 Displays one-variable and two-variable data in plots and summarises data with fractions, percentages, means and medians.	5.27 Reads and describes information in histograms, plots and summary statistics and reports on data collection processes and results.
See page 93	See page 93	See page 93
6.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, planning class intervals and fields collaboratively and individually.	6.26 Displays and summarises data to show location and variability (including when some grouping of data is required) to compare data sets and to show relationships in one data set.	6.27 Interprets collected and published data from tables, diagrams, plots, graphs, prose and databases to make comparisons and describe relationships.
See page 109	See page 109	See page 109
7.25 Compares, chooses and uses methods of organisation to suit the type of data and the questions asked.	7.26 Displays and summarises data to show location, variability and association, and links displayed data with measures of location, variability and association.	7.27 Selects and interprets information from collected and published data to construct arguments.
See page 125	See page 125	See page 125

OUTCOMES

ALGEBRA

Expressing generality	Function	Equations and inequalities	LEVELS
1.28-1.30 No outcomes at this level.			1
2.28-2.30 No outcomes at this level.			2
3.28-3.30 No outcomes at this level.			3
4.28-4.30 No outcomes at this level.			4
5.28 Follows and constructs rules for describing sequences and relationships between variable quantities using mostly natural language. See page 94	5.29a Generates and plots data in first quadrant coordinate graphs, describing patterns in the resulting scatter of points. 5.29b Informally sketches and interprets graphs that describe the relationship between two quantities in everyday situations. See page 94	5.30 Generates numbers or number pairs that satisfy a single constraint stated in natural language. See page 95	5
6.28 Uses basic conventions of algebraic notation in representing situations involving a variable quantity and explains why two linear expressions are equivalent. See page 110	6.29a Plots, sketches and interprets graphs, considering points, interval lengths, increases and decreases over an interval, and slope. 6.29b Recognises and represents at least linear and square relationships in tables, symbols and graphs and informally describes how one quantity varies with the other. See page 110	6.30 Sets up equations to represent one constraint in a situation, solves equations of the form $ax \pm b = cx \pm d$ and $ax^2 \pm b = c$ using guess, check and improve and graphical methods, and solves linear equations using analytic methods. See page 111	6
7.28 Uses algebraic notation in representing general properties of numbers and relationships between variables and establishes equivalence, including by using the distributive property and inverses of addition and multiplication. See page 126	7.29a Plots, sketches and interprets graphs in four quadrants, considering local and global features, including maxima and minima and cyclical changes. 7.29b Recognises and represents at least linear, reciprocal, exponential and quadratic functions in tables, symbols and graphs and describes assumptions needed to use these functions as models. See page 126	7.30a Sets up equations and pairs of simultaneous equations to represent constraints in a situation and solves, using guess, check and improve, graphical and, for linear equations, analytic methods. 7.30b Sets up inequalities to represent one or two constraints in a situation and generates a complete set of numbers or number pairs that satisfy the constraints. See page 127	7
8.28–8.30 Readily identifies algebraic form or structure in mathematical situations, recognising particular situations as instances of more general ones and moving readily between the general case and specific instances. See page 138			8

LEVEL ONE

LEVEL 1 Statement

At level one, students can use simple language to describe how shapes are alike or different. They attempt to show important spatial features when they draw or make things and have started to see a relationship between the shapes of things and the purposes to which they are put. They can match shapes even when required to change the orientation of a shape, can recognise pictures which do and do not have symmetry, and can use fold lines to produce symmetrical pictures. They can follow and use everyday language to give simple directions to describe position and can locate things in the classroom, school or home.

Students at this level can count, matching counting to objects and using numbers to order things. They can add and subtract small numbers when they occur in stories told to them, and can draw pictures and use materials to illustrate these stories. Using materials or calculators, they can explore and talk about simple number patterns. Without necessarily recognising particular coin values, they understand that money is used to buy things and that sometimes there is not enough money to buy something, while at other times they should get change.

Students can compare the size of two things and use everyday language to describe them. They can informally measure length and capacity by repeating units and can estimate to make judgements involving their own body. They use simple words to describe time and the passing of time and place things in sequence.

At this level, students are beginning to show that they recognise the element of chance in their lives. They can distinguish between certainty and uncertainty and understand that repetition of an action may produce different results. They can classify things according to familiar criteria, place collections in sequence and display data either physically or in pictures. With guidance, they can pose questions about these collections.

Students use simple language to describe ways their families use mathematics and, with some help, ask simple mathematical questions for themselves. They act out stories, depict questions with objects or use trial and error to answer questions. They are also starting to use some self-correcting behaviours.

WORKING MATHEMATICALLY

SPACE

NUMBER

The description for this strand extends over 4 pages

MEASUREMENT

CHANCE AND DATA

ALGEBRA

LEVEL 1
Table of outcomes

1.1 & 1.2 With guidance, poses questions that can in part be answered by classifying, matching, ordering or counting.		1.3 Answers mathematical questions by acting out a story, showing it with objects or pictures, or by trial and error.	1.4 Begins to exhibit some self-correcting behaviour in mathematical activities.	1.5 Talks about mathematical ideas in natural language.	1.6 Has some understanding of how mathematics is used in modifying social behaviour in classrooms and at home.
See page 22		See page 22	See page 23	See page 23	See page 23
1.7 Pays attention to shape when drawing or building things remembered, imagined, seen or handled.		1.8 Follows and gives oral directions based on everyday language of position and movement.	1.9 Talks about likenesses and differences between things seen or handled and begins to connect shape to function.	1.10 Repeats, reorients and turns over things when matching shapes and making pictures and patterns.	
See page 24		See page 24	See page 25	See page 25	
1.11 Counts and estimates collections, orders two or more collections, and orders things within collections.	1.12 Copies, continues and invents repeating and counting patterns and uses numbers to represent their patterns.	1.13 No outcome at this level	1.14a Represents self-generated or orally presented number stories involving small numbers using materials or drawings. 1.14b Understands that money is used in exchange for goods.	1.15 Uses counting and other means to mentally solve self-generated or orally presented questions from stories involving small numbers.	1.16 No outcome at this level	1.17 Uses a calculator to represent and explore numbers.
See page 26	See page 26		See page 27	See page 28		See page 29
1.18 Understands everyday comparative language associated with length, mass and capacity.	1.19 Directly compares and orders 'straight' lengths and capacities of two containers, including by repeating and counting units.	1.20 Makes non-numerical estimates of size involving everyday movements and actions.	1.21 Refers to familiar recurring phenomena when describing time.	1.22 No outcome at this level	
See page 30	See page 30	See page 31	See page 31		
1.23 Shows some recognition of the element of chance in familiar daily activities.	1.24 With guidance, poses questions about collected objects and information.	1.25 Participates in classifying and sequencing objects or pictures.	1.26 & 1.27 Displays objects and pictures and describes data in words and numbers.		
See page 32	See page 32	See page 33	See page 33		

1.28 – 1.30 No outcomes at this level

TOWARDS LEVEL 1

In TOWARDS LEVEL 1, level 1 pointers have been supplemented with additional pointers to show progress towards level 1 of students with disabilities. Level 1 outcomes and pointers start on page 22.

Working Mathematically

Investigating

Towards level 1, students:

- Demonstrate understanding of concepts such as behind, in front of, start, end, on top of, take away, more, less.
- Follow directions on the matching, classifying and ordering of objects.
- Follow directions or questions that use positional and locational words ('Put the milk in the fridge').
- Describe activities they have been engaged in.

Using problem-solving strategies

Towards level 1, students:

- Demonstrate understanding of everyday mathematical language (first, more, half).

Applying and verifying

Towards level 1, students:

- Use a variety of materials to make models.
- Work cooperatively in group investigations.
- Solve practical problems (getting on to a bus one at a time).

Using mathematical language

Towards level 1, students:

- Show understanding of everyday comparative language (big plate, small plate).
- Show understanding of solving simple number problems using concrete objects.
- Show understanding of winning a race (coming 1st, 2nd, 3rd).

Working in context

Towards level 1, students:

- Show an understanding of one-to-one correspondence in everyday experience (sharing fruit).

Space

Using spatial ideas, tools and techniques to interpret, draw and make

Towards level 1, students:

- Recognise attributes of familiar objects.
- Recognise shapes that are the same and different.
- Sort common objects according to shape (cutlery).
- Recognise and identify shapes in the world around them (traffic signs, toilet symbols).
- Experiment with shapes in a variety of materials (finger paint, folding washing).
- Construct a model with different materials (clay, dough, sand, cooking ingredients).
- Change position in games and movement activities.
- Match, recognise and label shapes.

Visualising, analysing and representing arrangements and locations

Towards level 1, students:

- Move their bodies through space (swimming, rolling, swinging).
- Follow simple directions (move from room to room in school, to bus-stop and back).
- Identify objects in familiar environments.
- Rearrange objects to suit a particular purpose (move desks so as to make room for another activity).

Visualising, analysing and representing shapes

Towards level 1, students:

- Assist in sorting, classifying, grouping and comparing of objects according to shape.
- Observe the function of objects.
- Demonstrate knowledge of the function of an object.
- Demonstrate functional knowledge of movement and directionality (fast, slow, up, down).

Visualising, analysing and representing movements and transformations

Towards level 1, students:

- Match real objects to a similar object.
- Match photographic or pictorial representations or symbols to objects.
- Recall parts of an object in sequential order.

Number

Count and order

Towards level 1, students:

- Recognise numerals needed for daily life (bus number, house and telephone numbers).
- Match numerals to familiar activities (TV on correct channel).
- Use strategies to simplify counting tasks (matching 3 dots to 3 apples when shopping).
- Estimate when events will occur (bus arrival, if bus is regularly used).

Number patterns

Towards level 1, students:

- Cooperate in games involving number patterns.
- Repeat rhymes, songs involving number patterns.
- Use number patterns in finding their way around (knowing that even house numbers are on the same side of the street).

Applying numbers

Towards level 1, students:

- Match items to people (put out one biscuit for each person present).
- Use terms such as all, many, few, a lot, a little.
- Understand differences in money value (packet of biscuits versus a television set).
- Initiate activities with coins, including handling, sorting, matching, changing, recognising in a variety of contexts.

Mental computation

Towards level 1, students:

- Demonstrate understanding of numbers used in books (using relevant concrete objects).
- Sing counting songs.

Calculators

Towards level 1, students:

- Identify calculator numbers.
- Use calculators to represent desired numbers.

Measurement

Choosing units

Towards level 1, students:

- Cooperate in activities that involve: matching, comparing, seriating; drawing, colouring, pasting, stamping; mopping, sweeping, digging, piling-up, hosing; pouring, filling, emptying, packing; balancing, throwing, rolling, carrying, cooking, modelling, shaping.
- Measure ingredients using arbitrary units including cups, spoons, handfuls, drops.

Measuring

Towards level 1, students:

- Cooperate in activities that involve building, constructing, arranging, stacking, sorting.
- Compare and contrast simple attributes of objects.
- Investigate attributes of familiar utensils, knives, forks.
- Use comparative language (big, small).

Estimating

Towards level 1, students:

- Cooperate in activities involving drawing, colouring, pasting, stamping, mopping, sweeping, digging, piling-up, hosing.

Time

Towards level 1, students:

- Refer to time of day, day of week, order of events in day and week in an informal way.
- Respond to music in a variety of movements including jumping, walking, dancing.
- Copy and repeat rhythmic patterns.
- Demonstrate understanding of relative time over short periods ('wait', 'soon').
- Recognise cues that signal beginning and end of activities.

TOWARDS LEVEL 1

Chance and Data

Understanding, estimating and measuring chance variation

Towards level 1, students:

- Assist group in representing arrangements of persons and objects.
- Assist group in representing by arrangements of pictures, symbols and paper tape.
- Demonstrate understanding of comparative language (maybe, will, won't).

Collecting data

Towards level 1, students:

- Assist group in games and activities involving frames, grids and diagrams.
- Collect and collate information and data (How many boys or girls in the class?).

Organising data

Towards level 1, students:

- Identify and name objects.
- Sort and classify familiar objects such as crockery.
- Sequence the day's routine (unpack bag, work, eat, go home).

Interpreting data

Towards level 1, students:

- Demonstrate 1:1 correspondence.
- Represent data by physical arrangement of people and objects.
- Represent data by pictographs and arrangements by symbols and paper tape.
- Collect and record data using informal tallies (shopping lists).
- Organise data in lists.
- Record data in a variety of forms.

Level 1 outcomes and pointers start on page 22.

LEVEL 1 Working Mathematically

Investigating

At level 1, a student:

1.1 & 1.2 With guidance, poses questions that can in part be answered by classifying, matching, ordering or counting.

Evident when students, for example:

- Ask 'why' or 'how' mathematical questions of other children (ask a friend why a coin has or has not been placed with those in that roll).
- Ask a mathematical question about a familiar situation ('We want to know how many are lost. How can we work that out?').
- Express their own mathematical questions ('We put the red shapes together and the green shapes together. I thought there were more red but Janey didn't. We wanted to know who was right.').
- Ask simple mathematical questions about children's stories ('How many children were in the family? Who was oldest?').
- Clarify a number question ('When we counted before there were seven books and now there are only three. Some must be lost, but we don't know how many.').

Conjecturing

Using problem-solving strategies

At level 1, a student:

1.3 Answers mathematical questions by acting out a story, showing it with objects or pictures, or by trial and error.

Evident when students, for example:

- Pretend and role-play in situations such as home, play, shops or imaginary worlds to answer questions involving ordering, counting, moving or placing (act out: 'The second person in the line had two people behind her. How many people in the line?').
- Act out a children's story involving position and orientation to answer questions about the story (act out *Bears in the Night* to decide when the boy arrives at certain places).
- Manipulate objects mentioned in a problem to find a solution (manipulate apples for a problem about apples, bottle tops for a problem about bottle tops).
- Draw a picture to represent a story involving a small numbers of things (draw a picture of a hen and chicks to work out how many chicks ran away).
- Suggest how they can answer their own mathematical questions ('If I line up the red and green shapes, I will be able to see if there are more green').

Applying and verifying

At level 1, a student:

1.4 **Begins to exhibit some self-correcting behaviour in mathematical activities.**

Evident when students, for example:

- Recognise some discrepancies in physical evidence (recognise a problem when they count 9 in the small pile and 7 in the larger pile).
- Respond to a direct challenge ('You and Pier have different results. Can you think why that might be?') by acknowledging there may be a problem, even if they are not sure what the problem is.
- Correct a pattern after noticing something wrong (notice and correct an error when making a one-two pattern by threading beads on a string).
- Self-correct their non-numerical estimates of size, (having estimated that they could reach the light switch and found they could not, adjust their estimates of other things they can and cannot reach).

Using mathematical language

At level 1, a student:

1.5 **Talks about mathematical ideas in natural language.**

Evident when students, for example:

- Describe how they solved a number problem ('We drew a picture of the 7 books we used to have and coloured in 3 books for the ones we still have. Then we counted the rest to see how many books were lost.').
- Participate in discussions to clarify the meaning of bipolar comparatives (to distinguish big–small from tall–short).
- Use the everyday language of movement and position when giving and following directions.
- Use the everyday language associated with ordinal and cardinal numbers to describe collections.
- Respond appropriately to and use simple comparative language (shorter–taller, heavy–heavier).
- Use terms of approximation (a bit more, about, just over, almost).

Working in context

At level 1, a student:

1.6 **Has some understanding of how mathematics is used in modifying social behaviour in classrooms and at home.**

Evident when students, for example:

- Say that by counting around the group they share the cubes fairly or they allocate turns fairly ('One potato, two potato...').
- Understand (using culturally relevant instances) that daily activities are related to time and that many of our activities are influenced by patterns in time (Ngaanyatjarra children use the order of daily phenomena to refer to time).
- Know that clocks' numbers and the position and movement of their hands reflect the passage of time, which in turn regulates aspects of our behaviour (say 'We have to wait until both hands move around to the twelve before we can go out for lunch').
- Identify some of the ways in which they and their families use numbers (their house and telephone number, deciding if there are enough plates).
- Identify some of the ways in which their lives are regulated by numbers and measurements (the cash register tells us how much to pay, the bus number tells us which bus to get on, the number in the fish and chip shop tells us which lot is ours).

Level 2 outcomes:

No outcomes at this level

Level 3 outcomes:

3.1 Poses mathematical questions prompted by similar or related questions or by specific stimulus materials.

3.2 Makes and tests conjectures including by responding to questions of the kind 'what would happen if...?'.

3.3 Uses problem-solving strategies that include those based on selecting key information and representing it in models, diagrams and lists.

3.4 Uses a variety of ways when prompted to check working and choice of method.

3.5 Integrates terms and notations from space, number, measurement and chance and data into comparisons and descriptions of things.

3.6 Describes some of the mathematics of own and other cultures, past and present.

LEVEL 1 Space

Using spatial ideas, tools and techniques to interpret, draw and make

At level 1, a student:

1.7 Pays attention to shape when drawing or building things remembered, imagined, seen or handled.

Evident when students, for example:

- Attend to general spatial features when making an object (select circular pieces for wheels, rectangular pieces for table tops).
- Attend to the essential spatial features of pictures or objects when copying them (copy a witch's hat made from triangles and a circle, choose suitable building blocks to make a house).
- Attend to shapes when making and drawing things from memory (draw a seesaw with a long flat board balancing on a point roughly at the centre).
- Draw or make a thing from an oral description which involves spatial language or implies shape ('Draw a tall hat with a pointy top' or 'Use plasticine to make a snake').
- Remember the shape of objects and parts of objects (select a shape most like a tree trunk or the ice cream on top of the cone).
- Convey the essential spatial features of common mathematical figures in their drawings (draw a shape that resembles a triangle with three roughly straight sides which more or less join).

Visualising, analysing and representing arrangements and locations

At level 1, a student:

1.8 Follows and gives oral directions based on everyday language of position and movement.

Evident when students, for example:

- Respond appropriately to and use the language of position and orientation (under, behind, in front of, below, on, alongside, near, right and left (carry out such requests as 'Put the dolly near the bears', say 'The book is under the desk').
- Respond appropriately to and use language of movement (back, forward, around, past, turn and up, say 'The bee flew around my head').
- Draw pictures or make things that illustrate the meaning of words relating to position and orientation (a picture with 'the crocodile behind the boy').
- Move self or objects to illustrate the meaning of words relating to position, orientation or movement (in drama, dance or physical activity).

Visualising, analysing and representing shapes

At level 1, a student:

1.9 Talks about likenesses and differences between things seen or handled and begins to connect shape to function.

Evident when students, for example:

- Interpret the words 'like' and 'different', describing how two things they can see and handle are alike ('These are the same colour') and how they are different ('But this is pointy and this isn't').
- Interpret and begin to use terms such as flat, straight, curved, side, round, corner (select tree trunks, posts and pencils to show 'long but round').
- Interpret and begin to use spatial terms such as rolls, slides, stacks to describe an object's function ('This could be used for the wheel because it rolls').
- Begin to connect shape and function in some familiar things (choose a triangular shape and reject a rectangular shape to rest a board on for a seesaw).
- Classify objects by a familiar attribute relating to shape, movement or function (good or not good for holding a drink).
- Recognise shapes close to rectangles, squares, triangles and circles in everyday things (on boxes and bicycles and on mathematical shapes such as cubes).

Visualising, analysing and representing movements and transformations

At level 1, a student:

1.10 Repeats, reorients and turns over things when matching shapes and making pictures and patterns.

Evident when students, for example:

- Match figures one to one (select a shape to match another and place it exactly on top to check).
- Fill in complex pictures and patterns where lines enable direct matching of component shapes (picture sticker books).
- Fill in pictures and patterns without dividing lines to enable direct matching of parts (complete a jigsaw puzzle).
- Make a copy of a complex picture, pattern or object when provided with a choice of parts.
- Fit figures and objects together based on shape and orientation.
- Use a fold line to produce symmetrical pictures by drawing freehand, folding, cutting, tracing.

Level 2 outcomes:

2.7a Fulfils simple spatial criteria when making things from verbal or visual descriptions.

2.7b Pays attention to the shape and placement of component parts when interpreting and making drawings.

2.8 Follows and gives descriptions of locations and paths and pays attention to order in reading and making informational maps.

2.9 Interprets common spatial language and, when prompted, uses it to describe the shape of things.

2.10 Generates patterns and follows rules based on the simple repetition and movement of things.

LEVEL 1 Number

Count and order

At level 1, a student:

1.11 Counts and estimates collections, orders two or more collections, and orders things within collections.

Evident when students, for example:

- Count collections of objects to answer the question 'How many are there?' and make or draw collections of a given size (respond correctly to 'Give me 6 bears').
- Make sensible estimates of the size of small collections up to 10 (for 7 buttons scattered on the table, 2 or 15 would not be a sensible estimate but 5 would be reasonable).
- Skip count in 2s or 3s using a number line, hundred chart or mental counting (2, 4, 6...).
- Use numbers to decide which collection is bigger, smaller, same size ('If he's got 7 mice at home and I only have 5, then he has more').
- Use the terms first, second, third to indicate position in a sequence ('I finished my lunch second', 'The red bear is fourth') and make or draw a sequence to represent ordinal numbers (place the pink bear so it is number 3 —or third— in the row).
- Pretend and role-play with other children to answer questions involving ordering and counting (work with their group to act out 'The second person in the line had two people behind her. How many people in the line?').
- Combine and separate collections to represent small numbers, say 7, in different ways (7 is 3 and 4 more, it is also 6 and 1 more).

Number patterns

At level 1, a student:

1.12 Copies, continues and invents repeating and counting patterns and uses numbers to represent their patterns.

Evident when students, for example:

- Copy and continue in materials the pattern of counting numbers or skip counting in 2s or 3s (make and continue a staircase that starts 1 cube, 2 stacked cubes, 3 stacked cubes; copy by threading 2 red on to the first rod, 2 red and 2 green on to the next, 2 red, 2 green and 2 yellow on the next, and continue the pattern in colours of their own choice).
- Copy and continue repeating patterns (a necklace with the sequence two gum leaves, three gum nuts, one shell, two gum leaves, three gum nuts, one shell).
- Invent their own repeating patterns in a range of media (use a xylophone to make up a four note tune and repeat over and over).
- Repeat a pattern in a different medium (show the necklace pattern in sounds or the xylophone pattern in colours).
- Use simple language such as 'pattern', 'over and over', 'repeat', 'again' to describe patterns and to say what is the same about two versions of the same pattern (the necklace goes 2 leaves, 3 nuts, 1 shell, over and over; they both go 2, then 3, then 1).
- Use a sequence of numbers to represent a repeating pattern (write 2, 3, 1, 2, 3, 1, 2 ... as another way to show the leaves, nuts and shell pattern).
- Make up representations of a repeating or counting number pattern (3, 3, 3, 3.../1, 3, 1, 3, 1, 3... /1, 2, 3, 4... or 2, 4, 6, 8 ...).

Equations

1.13 **No outcome at this level in this sequence.**

Applying numbers

At level 1, a student:

1.14a Represents self-generated or orally presented number stories involving small numbers using materials or drawings.

Evident when students, for example:

- Can think of number questions about stories read to them ('How many children were in the family?', 'Who was oldest?').
- Manipulate the objects in a problem to find a solution (manipulate felt pens for a problem about felt pens, bottle tops for a problem about bottle tops).
- Use objects to represent a number story (after being told a story about black kittens and ginger kittens, they represent the kittens with two colours of buttons).
- Act out a number story to answer questions (children take on the role of the kittens and act out the story to decide how many there were altogether).
- Draw a picture to represent a story involving a small numbers of things (draw a picture of a hen and chicks to work out how many chicks ran away).
- Combine and separate collections using a variety of materials and express the results orally or in pictures ('I took 8 blocks from the bag. I had 3 red ones and 5 blue ones. I had 4 thick ones and 4 thin ones.').

1.14b Understands that money is used in exchange for goods.

Evident when students, for example:

- Offer play money in exchange for goods in the classroom shop and put prices on items when playing.
- Show that they know that money is needed to buy things (say that they don't have any money for an icecream).
- Sort coins and notes and realise that they have different values.
- Know that some items cost more than others (say, 'That will probably cost more').
- Know that change is expected if too much money is given and that if too little money is offered something cannot be bought (ask, 'Do I have enough?').

LEVEL 1 Number

Mental computation

At level 1, a student:

1.15 Uses counting and other means to mentally solve self-generated or orally presented questions from stories involving small numbers.

Evident when students, for example:

- Use their own mental counting strategies to add and subtract small numbers that are part of stories ('Rachel found 3 eggs under the first hen, then she found 2 more under the next hen. How many has she so far? What if she found 3 more?').
- Mentally add and subtract small numbers in stories.
- Decompose numbers and represent them in a variety of ways to assist in adding and subtracting (think of 5 and 3 more as the same as 2 fours).
- Find hidden numbers mentally with sums to at least 10 (watch as 7 dinosaurs are counted into a box and as 5 are removed one at a time, and say how many are left in the box).

Written computation

1.16 No outcome at this level in this sequence.

Calculators

At level 1, a student:

1.17 Uses a calculator to represent and explore numbers.

Evident when students, for example:

- Investigate the order of entering digits and work out that they need to enter the digits in order from left to right (enter 14 by pressing 1 then 4).
- Count by adding 1 each time, beginning with 0 and press [+] [1] [=] repeatedly or [+] [1] [=] [=] [=] in order to count (make calculator display change from 5 to 6).
- Find ways to make the calculator count by twos or threes or any number each time.
- Use the calculator to count a collection one at a time (add 1 to their score in the calculator as each child who has a banana brings it to the front of the room).
- Count a collection two or three at a time (for each child in the group, the calculator operator adds two to the score on the calculator; the children then count the eyes in their group to check the total).

Level 2 outcomes:

2.11 Counts, orders, estimates and describes with whole numbers within everyday experience and with fractions expressed in words.

2.12 Identifies, continues and invents whole number patterns, including where successive terms in a sequence can be linked by addition or subtraction of a constant amount.

2.13 Uses understanding of whole numbers and addition and subtraction relationships to construct and complete simple statements of equality.

2.14a Represents problems involving the four basic operations on whole numbers in a variety of ways, using stories, materials, pictures and number sentences.

2.14b Compares prices with money available, calculates the exact and approximate cost of several items and makes change.

2.15 Estimates and calculates mentally, including adding and subtracting numbers to 10 and making extensions based on place value.

2.16 Uses a variety of strategies, including regrouping, to assist in adding and subtracting whole numbers when unable to complete mentally.

2.17 Uses a calculator to represent and explore numbers, place value and operations.

LEVEL 1 Measurement

Choosing units

At level 1, a student:

1.18 Understands everyday comparative language associated with length, mass and capacity.

Evident when students, for example:

- Respond appropriately to and use comparative language of length such as shorter than, taller than, longer than, same length, narrow/wide, near/far.
- Respond appropriately to and use for themselves comparative language such as heavier-lighter, weighs more-weighs less, too heavy.
- Respond appropriately to a request to find a container that will hold more or hold less or hold about the same.
- Use comparative language such as holds more or holds less to describe the capacity of clearly different containers.
- Choose the heavier of two different objects by hefting and find objects heavier than, lighter than, about the same as a given object.
- Make a thing from an oral description relating to size ('Make a long, thin snake', 'Make the road wider').

Measuring

At level 1, a student:

1.19 Directly compares and orders 'straight' lengths and capacities of two containers, including by repeating and counting units.

Evident when students, for example:

- Select something longer than, shorter than, or the same length as something else by matching (compare their twig with others, sorting into those that are too long, too short and about right).
- Use body parts (hand spans, foot lengths, strides) and familiar objects (leaves, pop sticks) repeatedly as a unit and count units to compare the length of two things.
- Line up an end of things to enable a direct comparison of length.
- Use indirect methods to range by length objects which can't be directly compared (fit tape along curved paths and range the paper tapes to decide which path is longest).
- Respond to a request to convince someone which of two clearly different containers holds more by pouring from one to the other.
- Use such measures as spoons, scoops and cups repeatedly and say how many units will fill a container.

Estimating

At level 1, a student:

1.20 Makes non-numerical estimates of size involving everyday movements and actions.

Evident when students, for example:

- Make personal estimates in moving and acting (say 'I think I am tall enough now to reach that light switch', 'This will be too heavy for me to lift').
- Find things clearly bigger than or smaller than a given object.
- Guess whether something will be longer than, shorter than, same length as a given thing and check by matching (when making a mobile, select a piece of tape they think will be longer than one used before).
- Guess how many times a provided or chosen object will fit along another object, then check.
- Use appropriately everyday language of approximation (about, almost, nearly, not quite, just under, a bit under).

Time

At level 1, a student:

1.21 Refers to familiar recurring phenomena when describing time.

Evident when students, for example:

- Respond appropriately to and use the local comparative and descriptive language of time (before-now-after, longer-shorter, day-night, high tide, summer, the wet).
- Order events within today ('Today I woke up, got dressed, had breakfast...') and place regular activities in typical order (the stages of having a bath or of catching and cooking a fish).
- Relate days of the week and months of the year to events in their lives (birthdays, holidays, school days and weekends, Easter, Ramadan).
- Count informal units of elapsed time (use claps, turns or drumbeats to measure the time taken to complete a task).
- Understand that clocks are used to tell the time of day and link specific times on a clock to recurring daily phenomena (getting light, daytime, getting dark, night-time).
- Understand that clocks are used to reflect the passage of time, linking the movement of hands or the changing of numbers to the passage of time.

Using relationships

1.22 No outcome at this level in this sequence.

Level 2 outcomes:

2.18 Chooses the appropriate physical attribute when comparing and measuring things and units which relate to that attribute.

2.19 Directly and indirectly compares lengths and capacities, and measures and makes lengths and distances by counting uniform units, including metres and centimetres.

2.20 Estimates the order of things by length, area, mass and capacity and makes numerical estimates of length using a unit that can be seen or handled.

2.21 Locates and sequences events in time, beginning to read familiar clocks and calendars.

2.22 No outcome at this level

LEVEL 1 Chance and Data

Understanding, estimating and measuring chance variation

At level 1, a student:

1.23 Shows some recognition of the element of chance in familiar daily activities.

Evident when students, for example:

- Respond appropriately to everyday language associated with uncertainty (will-won't-might, could-couldn't).
- Talk about events in ways which show they recognise their chance nature (say, 'Our new baby might be a girl or it might be a boy').
- Use language such as 'won't happen', 'will happen' or 'might happen' appropriately (says, 'Tomorrow it might rain' or 'I need seven to win and that can't happen').
- Recognise and take into account the possibility of different results for repetitions of the same simple action, such as throwing a die for a game (say, 'I got 3 last time but I won't always').

Collecting data

At level 1, a student:

1.24 With guidance, poses questions about collected objects and information.

Evident when students, for example:

- Follow the modelling of questions by their teacher ('What biscuit is most common? How many of that biscuit are there?'), ask similar questions about collected objects or pictures.
- Offer questions when prompted by their teacher's questions, such as, 'What questions does Mario's news make you think of?', 'Are we all the same age?', 'Has anyone else caught a fish?'
- Offer suggestions about what they could collect to answer their own questions or those posed by the teacher (place favourite colour pen on their desk, collect a leaf from their favourite tree, write their saint's day on a card, draw a picture of the last animal they saw).

Organising data

At level 1, a student:

1.25 Participates in classifying and sequencing objects or pictures.

Evident when students, for example:

- Describe a likeness between several things (all leaves, trees and pencils are alike because they are long and round) or a difference between several things (some are leaves but some are not).
- Classify objects using one or two familiar criteria (leaves by shape and colour, list class members' names under Aboriginal kinship group).
- Place objects into sequences (order leaves according to criteria such as darkness or length).
- Suggest how they can answer questions about their collections ('If I line up the red and green shapes, I will be able to see if there are more green...').

Displaying and summarising data

At level 1, a student:

1.26 & 1.27 Displays objects and pictures and describes data in words and numbers.

Evident when students, for example:

- Display classified objects to compare collections (string red beads and green beads separately to decide which collection has more).
- Display in one to one correspondence pictures or objects that represent themselves (place in rows— one each for car, walk, bus— pictures that show how they came to school; make physical block graphs from interlocking cubes where each colour represents their kinship group).
- Draw a picture as a record of their results (draw a picture of the graph they made with real pieces of fruit).
- Summarise information by counting (count how many of children have caught a fish and say '11 children have caught a fish').
- Talk about what they have found from their data collection and display ('We lined up the fruit in rows. There were more apples than bananas.').

Interpreting data

Level 2 outcomes:

2.23 Distinguishes possible from impossible events, and describes familiar and easily understood events as more likely or less likely to happen.

2.24 Contributes questions in group activities and realises some can be answered by collecting data in the form of objects or information.

2.25 Contributes to deciding how to classify and sequence data, applying unambiguous and familiar criteria consistently.

2.26 Displays and summarises data based on one-to-one correspondence between data and representation.

2.27 Describes, orally and in writing, what own and classmates' displays of data show.

LEVEL TWO

LEVEL 2 Statement

Students at level two are able to draw and make things that meet simple requirements of shape and structure. When describing objects, they refer to shape rather than other features. They know the names of some common shapes and objects and can discriminate between them. Students are beginning to develop some of the skills needed for constructing objects from their parts. They can make patterns from repeating and moving shapes according to simple rules and can put key features in the right order in simple maps. They can also find objects and describe paths on simple grids.

At this level, students can count with at least two digits and estimate collections up to a size of 20 without counting. Although not using symbols, they understand the fractions one-half, one-third and one-quarter and can divide things into these parts. They not only understand addition and subtraction, but they can create story problems and recognise that subtraction is useful for different types of problems, such as direct subtraction and the difference between. They use simple rules to make number patterns and identify and explain rules involving constant addition and subtraction. They remember many basic addition and subtraction facts and have strategies to help them complete other additions and subtractions. They can subtract two digit numbers using materials, diagrams, calculator or paper-and-pencil methods. They are beginning to understand what multiplication and division are. They recognise the value of Australian coins and decide whether they have enough money to buy items. They can calculate the exact and approximate cost of several items and determine change.

Students at this level focus on the correct qualities when comparing things. They make estimates based on units they can see or handle, measure carefully using informal units, and use a metre rule or tape to measure length in metres and centimetres. They are developing a stronger sense of the passage of time and can order familiar events in their lives into typical sequences according to time of the day or year. They have begun to use calendars and tell time on a clock.

Students are also beginning to get a sense of some things being more likely than others and can understand that some things are impossible. They ask some mathematical questions in group discussions and make simple predictions. They use a variety of ways, including pictures or simple bar graphs, to summarise and display what they have found and can describe their own collections and those of others in speech and writing.

WORKING MATHEMATICALLY

SPACE

NUMBER

The description for this strand extends over 4 pages

MEASUREMENT

CHANCE AND DATA

ALGEBRA

LEVEL 2
Table of outcomes

2.1 – 2.6 No outcomes at this level

2.7a Fulfils simple spatial criteria when making things from verbal or visual descriptions.

2.7b Pays attention to the shape and placement of component parts when interpreting and making drawings.

See page 38

2.8 Follows and gives descriptions of locations and paths and pays attention to order in reading and making informal maps.

See page 38

2.9 Interprets common spatial language and, when prompted, uses it to describe the shape of things.

See page 39

2.10 Generates patterns and follows rules based on the simple repetition and movement of things.

See page 39

2.11 Counts, orders, estimates and describes with whole numbers within everyday experience and with fractions expressed in words.

See page 40

2.12 Identifies, continues and invents whole number patterns, including where successive terms in a sequence can be linked by addition or subtraction of a constant amount.

See page 40

2.13 Uses understanding of whole numbers and addition and subtraction relationships to construct and complete simple statements of equality.

See page 41

2.14a Represents problems involving the four basic operations on whole numbers in a variety of ways, using stories, materials, pictures and number sentences.

2.14b Compares prices with money available, calculates the exact and approximate cost of several items and makes change.

See page 41

2.15 Estimates and calculates mentally, including adding and subtracting numbers to 10 and making extensions based on place value.

See page 42

2.16 Uses a variety of strategies, including regrouping, to assist in adding and subtracting whole numbers when unable to complete mentally.

See page 42

2.17 Uses a calculator to represent and explore numbers, place value and operations.

See page 43

2.18 Chooses the appropriate physical attribute when comparing and measuring things and units which relate to that attribute.

See page 44

2.19 Directly and indirectly compares lengths and capacities, and measures and makes lengths and distances by counting uniform units, including metres and centimetres.

See page 44

2.20 Estimates the order of things by length, area, mass and capacity and makes numerical estimates of length using a unit that can be seen or handled.

See page 45

2.21 Locates and sequences events in time, beginning to read familiar clocks and calendars.

See page 45

2.22 No outcome at this level

2.23 Distinguishes possible from impossible events, and describes familiar and easily understood events as more likely or less likely to happen.

See page 46

2.24 Contributes questions in group activities and realises some can be answered by collecting data in the form of objects or information.

See page 46

2.25 Contributes to deciding how to classify and sequence data, applying unambiguous and familiar criteria consistently.

See page 47

2.26 Displays and summarises data based on one-to-one correspondence between data and representation.

See page 47

2.27 Describes, orally and in writing, what own and classmates' displays of data show.

See page 47

2.28 –2.30 No outcomes at this level

LEVEL 2 Space

Level 1 outcomes

1.7 Pays attention to shape when drawing or building things remembered, imagined, seen or handled.

1.8 Follows and gives oral directions based on everyday language of position and movement.

1.9 Talks about likenesses and differences between things seen or handled and begins to connect shape to function.

1.10 Repeats, reorients and turns over things when matching shapes and making pictures and patterns.

Using spatial ideas, tools and techniques to interpret, draw and make

At level 2, a student:

2.7a Fulfils simple spatial criteria when making things from verbal or visual descriptions.

Evident when students, for example:

- Make things that meet criteria or needs relating to function (will roll, will stack, will stand up by itself, can remove a piece to make a door).
- Make constructions from visual instructions (as used for children's construction toys).
- Identify mathematical shapes such as cones or rectangles on which familiar things are based.
- Select materials and methods to achieve the ends they have in mind (select objects that will stack to build a model of the walls of their house from memory).
- Explain, in terms of a given description, how they could improve things they have made.

2.7b Pays attention to the shape and placement of component parts when interpreting and making drawings.

Evident when students, for example:

- Correctly draw figures showing the essential spatial features of named mathematical shapes (when asked to draw a pentagon, draw five straight sides that join end to end to form a closed curve).
- Copy pictures composed of simple geometric figures so that the main components are recognisable in shape, position and orientation (make a freehand copy of a picture of a table setting using the same mathematical shapes in roughly the same positions and orientations).
- Draw from memory an arrangement of several shapes, either freehand or with a template (look at an arrangement of four shapes made by their partner before covering and making a sketch of it).
- Select a set of 2D figures from a collection to match the faces of a provided 3D shape.

Visualising, analysing and representing arrangements and locations

At level 2, a student:

2.8 Follows and gives descriptions of locations and paths and pays attention to order in reading and making informal maps.

Evident when students, for example:

- Place important things in their environment in order on their map (taps, toilet and shady trees in school yard).
- Describe the location of an object as between two other locations ('Mummy's work is between our house and school because we drop her on the way').
- Follow and give directions using language of position ('second from the left', 'three spaces up and two to the right', 'inside').
- Find paths on informal maps and mazes (a path between two parts of the playground or through computer-generated mazes).
- Find and explain paths given starting and finishing points and criteria such as safest, quickest, driest.
- Give instructions to create paths on squared paper or computer screens ('Forward 3, turn right, forward 7, turn left').

Visualising, analysing and representing shapes

At level 2, a student:

2.9 Interprets common spatial language and, when prompted, uses it to describe the shape of things.

Evident when students, for example:

- Respond to requests to talk about the shape of an object, place or mathematical model, using language such as flat, curved, corner, side, round, square, edge ('Tell me about the shape of the daisy').
- Identify figures in faces and cross-sections of objects (draw and, where a common name is available, name six different figures in a jug).
- Distinguish rectangles, including squares, and triangles and circles from other figures that resemble them (describe some of their salami slices as circular but others as not and explain that some slices are not circles because they are longer one way).
- Identify and name cubes, rectangular boxes, cylinders, cones and spheres in common things (say, 'If I remove the handle from this mug, it looks like a cylinder').
- Distinguish between 3D object and its face (know a cube is not a square and a box is not a rectangle).
- Sort things according to simple spatial criteria and describe and represent classifications including in two-way tables (classify objects by how many faces they have and the shape of the faces).

Visualising, analysing and representing movements and transformations

At level 2, a student:

2.10 Generates patterns and follows rules based on the simple repetition and movement of things.

Evident when students, for example:

- Understand two cut-out figures as exactly the same (same shape and size) if one fits on the other exactly and classify cut-out figures into groups that fit exactly on top of each other.
- Explain their choice of simple figures which are the same with language such as turn, turn over, slide along ('We thought the tall rectangle was different from the wide one, but we turned the tall one around and saw we could slide it on top of the wide one').
- Repeat shapes which are exactly the same in some recognisable pattern for decorative purposes (select three types of pattern block and use multiple copies to make a design).
- Describe the unit of change (such as a flip or turn) in a spatial sequence (we started with ~ and each time we flipped the shape over once and moved along one space).
- Make symmetrical pictures by a variety of means including using cut-out shapes and by flipping and drawing around templates.
- Use multiple copies of a 3D object to decide whether repetitions will stack or pack and make repeating patterns with those that will stack or pack.

Level 3 outcomes:

3.7a Pays attention to the shape and placement of parts when matching, making and copying things, including matching nets with 3D shapes.

3.7b Matches actual models and conventional drawings of them and attends to what can be seen when making drawings.

3.8 Visualises, follows and gives descriptions of locations and paths and attends to order and proximity in reading and making maps.

3.9 Interprets common spatial language and uses it to describe and compare features of things.

3.10 Recognises and uses repetitions and movements of the same shape embedded within arrangements and patterns.

LEVEL 2 Number

Level 1 outcomes:

1.11 Counts and estimates collections, orders two or more collections, and orders things within collections.

1.12 Copies, continues and invents repeating and counting patterns and uses numbers to represent their patterns.

1.13 No outcome at this level

1.14a Represents self-generated or orally presented number stories involving small numbers using materials or drawings.

1.14b Understands that money is used in exchange for goods.

1.15 Uses counting and other means to mentally solve self-generated or orally presented questions from stories involving small numbers.

1.16 No outcome at this level

1.17 Uses a calculator to represent and explore numbers.

Count and order

At level 2, a student:

2.11 Counts, orders, estimates and describes with whole numbers within everyday experience and with fractions expressed in words.

Evident when students, for example:

- Count forwards and backwards from any whole number, including skip counting in 2s, 3s and 10s.
- Express their conviction that they should get the same answer each time they count a particular collection, regardless of the strategy used.
- Use place value to distinguish and order whole numbers (write four ten dollar notes and three one dollar coins as $43 and say why $34 is not correct, explain why 413 children is more than 279 children by referring to the hundreds place).
- Estimate the size of a collection (up to about 20) by mentally or visually grouping the items or comparing the group with one of known size ('That jar has 100 beans in it and I think this has more, so I guess about 120').
- Use fractional language (one half, third, quarter, fifth, tenth) appropriately in describing and comparing things ('I ate about a quarter of my Easter egg but you ate about half').
- Separate objects and collections into equal parts to show and compare unit fractions (find a third of a cup of sugar, a quarter of the dough, show that half the pens is more than a third of them).
- Describe and record simple fractional equivalences in words (say, 'We found that the left over half pizza was as much as our two quarters put together' and write 'one quarter = two halves').

Number patterns

At level 2, a student:

2.12 Identifies, continues and invents whole number patterns, including where successive terms in a sequence can be linked by addition or subtraction of a constant amount.

Evident when students, for example:

- Represent simple adding and subtracting patterns visually, choosing materials to help make the pattern obvious (represent 21, 31, 41, 51... by grouping materials to show the tens in some way).
- Write a pattern of numbers to fit a matchstick pattern and use to make predictions (predict the number of matches needed to make a set of five triangles. What about ten triangles? Explain why.).

- Describe a rule that could have been used to generate an adding or subtracting sequence and test the rule by using it ('To make that sequence you could start with 50 and take 3 each time, so that would be 50, 47, 44, 41, 38... which is what you had').
- Continue or fill in number sequences involving addition or subtraction by a constant amount (fill in 15, 25 ... 45, 55).
- Identify one-stage rules such as 'add 3', 'subtract 5' or 'double' when playing guess-my-rule games.
- Describe patterns in lines in a hundred chart and explain with an adding or subtracting rule ('To get that line, start with 2 and add 11 each time' or 'Start with 31 and take 9 each time').
- Understand that sequences of multiples can be generated by adding a constant amount from zero (the multiples of 3 are 3, 6, 9 ... but not 4, 7, 10...).
- Understand the difference between odd and even numbers and know that they can be distinguished by observing the last digit.

Equations

At level 2, a student:

2.13 Uses understanding of whole numbers and addition and subtraction relationships to construct and complete simple statements of equality.

Evident when students, for example:

- Generate equations involving addition and subtraction given a simple constraint ('A student called out 18. The teacher said, "That's right". What could the question have been? What other questions can you think of?').
- Order sets of related additions or subtractions to generate additional facts based on patterns (use 1 + 14, 2 + 13, 3 + 12 to conjecture other sums to 15).
- Use addition and subtraction relationships and place value to write related equations (4 + 5 = 9, so 4 = 9 – 5 and 5 = 9 – 4; 6 + 7 = 13, and 16 + 7 will be ten more, so 16 + 7 is 23).
- Use their knowledge of place value to conjecture about hidden numbers (say that they think 30 is hidden in [ ] + 13 = 43 because 43 is four tens and three ones, so three tens must be hidden).
- Test their conjectures about equivalent statements and about hidden numbers (by showing 30 + 13 with materials or with a calculator) and try to work out where their thinking went astray when their predictions prove wrong.
- Explain to peers the meaning of questions from written mathematical material (explain that in the instruction 'Complete: 7 + [ ] = 27', the question is 'What number added to 7 gives 27?').

Applying numbers

At level 2, a student:

2.14a Represents problems involving the four basic operations on whole numbers in a variety of ways, using stories, materials, pictures and number sentences.

Evident when students, for example:

- Interpret expressions such as less than, what's left, twice as many as, how many times.
- Identify a symbolic expression with alternative verbal expressions (12 – 5 can be said as 'twelve take away five', 'twelve subtract five' or 'the difference between 12 and 5'; 45 ÷ 5 = ? may be read 'What is 45 divided by 5?' or 'How many lots of 5 in 45?').
- Read and write simple numerical statements involving the four operations, given that simple means those with essentially one component as in 36 + 17 = 53 but not 8 + (11 – 7) = 12.
- Select an appropriate addition or subtraction of whole numbers within their experience to deal with combining, taking away and difference.
- Use materials and diagrams to represent multiplication and division expressed in words (draw a picture to represent 'If 1 bike has 2 wheels and 2 bikes have 4 wheels, how many wheels would 8 bikes have?').
- Write 'stories' to represent operations expressed symbolically (write a problem which could be solved by finding 35 + 16 or 3 x 5).

2.14b Compares prices with money available, calculates the exact and approximate cost of several items and makes change.

Evident when students, for example:

- Count coins, including multiples of 5c, 10c, 20c, 50c, $1 and $2, and record total amounts.
- Read amounts of money and make up the amount with coins in different ways.
- Give change using a counting on with coins strategy.
- Decide whether they have more or less money than the price of something and whether to expect change ('The lunch cost $2.85, so I gave the shop assistant $3. He will have to give me some back.').
- Enter, add and subtract amounts of money in a calculator and interpret the calculator displays.
- Round up to decide whether they have enough money to buy something (decide that two items worth 48c and 37c will together cost less than a dollar because each is less than 50c).

LEVEL 2 Number

Mental computation

At level 2, a student:

2.15 Estimates and calculates mentally, including adding and subtracting numbers to 10 and making extensions based on place value.

Evident when students, for example:

- Remember many basic addition facts and work out the others ('I don't know 7 + 9, but 7 + 7 is 14 and this will be two more').
- Use counting strategies such as combining collections and counting the lot, counting on from the first number, counting on from the largest number (to find 16 + 9, start with 16, hold up 9 fingers and count on, 17, 18, 19... as they fold down each finger).
- Use their understanding of the relationship between addition and subtraction to work out subtractions ('If I take 7 from 13, there must be 6 left because 6 + 7 is 13, so 13 – 7 = 6').
- Count forwards and backwards in tens from a one-or two-digit starting point (16, 26, 36 ...).
- Use their understanding of place value to make mental extensions of addition facts (6 add 7 is 13, so 16 add 7 is 23; 7 take 4 is 3, so 70 take 40 is 30).
- Estimate sums by rounding to single digit numbers (consider 'If one book costs $3.95 and the other $5.25, will $10 be enough for the two?' and say that the total must be less than $4 add $6).

Written computation

At level 2, a student:

2.16 Uses a variety of strategies, including regrouping, to assist in adding and subtracting whole numbers when unable to complete mentally.

Evident when students, for example:

- Use materials and diagrams to solve addition and subtraction word problems (show 'Our holiday lasts 3 weeks and we've already had 9 days, how many days are there to go?' with 3 bundles of 7 sticks and 9 removed).
- Regroup money to assist in adding (to find the cost for lunch, show the value of the items with coins, combine the coins and regroup into dollars).
- Use materials and diagrams to solve symbolically expressed additions and subtractions (represent 24 + 37 with MAB material or with play money in dollars and trade to solve the problem).
- Use informal written methods based on place value to solve addition and subtraction problems (write 24 + 37 as 20 + 4 + 30 + 7, rewrite as 50 + 11 = 61).
- Round sensibly to estimate (round each of $19 and $18 to $20 to decide if $40 will be enough to buy both).
- Decide whether their sum or difference is sensible by estimating ('19 + 18 will be pretty close to 40, so I think 27 is wrong').

Calculators

At level 2, a student:

2.17 Uses a calculator to represent and explore numbers, place value and operations.

Evident when students, for example:

- Count by twos (threes...) on a calculator (keep on adding four by pressing [+] [4] [=] each time or by [+] [4] [=] [=] [=]).
- Test a conjecture about sets of equivalent numerical expressions where the computational demands go beyond their current skills (test their conjecture that 18 + 12 = 30, so 118 + 12 = 130).
- Use the calculator to test their own understanding of place value (write a number with no digit greater than 8 — for example, 374 — increase the smallest digit by 1, the next smallest by 1, then the next).
- Recognise that the subtract key can be used for a variety of different situations involving taking away, comparison or difference.
- Use the calculator to link operations (find 5 + 5 + 5 + 5 =, 16 + 16 =, 9 + 9 + 9 =, 36 + 36 + 36 =, if the [+] key is broken).
- Add and subtract any numbers arising in daily activities (from class numbers, calculate how many students in the school).

Level 3 outcomes:

3.11 Counts, orders, estimates and describes with whole numbers, common fractions and decimal fractions for money or measurements (two places).

3.12 Identifies, continues and invents whole number patterns involving the four operations, including where successive terms in a sequence can be linked by an addition or subtraction strategy.

3.13 Uses understanding of whole and fractional numbers, relationships and operations to construct and complete simple statements of equality.

3.14 Makes a suitable choice of operation for situations involving whole numbers, amounts of money and familiar measurements.

3.15 Estimates and calculates mentally, including adding (sum to 100) and subtracting two-digit numbers and multiplying numbers to 10.

3.16 Uses understood written methods to add and subtract any whole numbers and amounts of money and to multiply and divide whole numbers by whole numbers to 10.

3.17 Uses a calculator for operating on whole numbers, amounts of money and measurements.

LEVEL 2 Measurement

Level 1 outcomes:

1.18 Understands everyday comparative language associated with length, mass and capacity.

1.19 Directly compares and orders 'straight' lengths and capacities of two containers, including by repeating and counting units.

1.20 Makes non-numerical estimates of size involving everyday movements and actions.

1.21 Refers to familiar recurring phenomena when describing time.

1.22 No outcome at this level

Choosing units

At level 2, a student:

2.18 Chooses the appropriate physical attribute when comparing and measuring things and units which relate to that attribute.

Evident when students, for example:

- Clarify the attribute to be used for ordering things by physical size (say, 'By bigger do you mean taller or wider or which one holds the most?').
- Choose the attribute to be used for ordering (say, 'We had to decide which were the best juice containers, so we decided to see which holds most').
- Select or reject units as relating or not relating well to the attribute to be measured (use cupfuls of rice to measure the capacity of a container, reject a piece of string when area is the quantity of interest).
- Attend to the right attribute (of length, area, volume, mass) when directly comparing things (pick the heavier of a lump of fruitcake and a piece of wood by picking each up and hefting rather than choosing on the basis of visual size).
- Understand that if different attributes are used to order objects, the resulting ordering may be different (explain that the tallest glass may not hold the most because it is not as wide as the others).

Measuring

At level 2, a student:

2.19 Directly and indirectly compares lengths and capacities, and measures and makes lengths and distances by counting uniform units, including metres and centimetres.

Evident when students, for example:

- Use everyday language to compare distances ('The gum tree is further away than the big rock', 'My route to the shop is shorter than yours').
- Compare the capacity of two differently shaped containers by pouring from one to the other or by pouring from each into two identical containers.
- Use a unit consistently and carefully to measure length (butt the same size paper clips next to each other when measuring the length of a book).
- Use a unit consistently and carefully to measure and compare containers (estimate which storage dish holds more and test by carefully counting how many level cupfuls of beans fill each).
- Express their measurements using between statements or with parts of a unit (the hallway is between 9 and 10 books wide, the jug holds 5 full cups and about another third).
- Use a metre rule, ruler or tape measure correctly to construct things to a specified length in metres or in centimetres or in metres and centimetres.

Estimating

At level 2, a student:

2.20 Estimates the order of things by length, area, mass and capacity and makes numerical estimates of length using a unit that can be seen or handled.

Evident when students, for example:

- Attend to the right attribute when judging which of two things is bigger (attend to area rather than length when asked to say which mat would cover more of the floor).
- Judge by looking which containers will hold about or more than or less than a given container and by hefting which objects will be about the same, heavier or lighter than a given object.
- Judge which straight and curved things they can see have a length of about or less than or more than a given thing (including a one metre or one centimetre length).
- Estimate distance in a familiar unit such as paces, and show improvement in their estimates as a result of pacing out to measure the distances.
- Estimate the number of times a unit of length they can handle will fit along an object and show improvement in their estimates as a result of testing.
- Show some improvement in their judgements of the order of length, area, capacity and mass as a result of testing (use a balance beam).

Time

At level 2, a student:

2.21 Locates and sequences events in time, beginning to read familiar clocks and calendars.

Evident when students, for example:

- Identify recurring phenomena that act as personal or local community markers of time of day or year (the wet, shearing, a birthday or saint's day, shift changeover, church bells).
- Locate regular activities in time (produce a simple timeline of what they usually do on Saturdays).
- Order times of day and year by natural events (the ground becomes clear, the first light, sunrise, early morning) or by cultural events (New Year, school starts, Easter and school holiday).
- Estimate the time of day, using natural or artificial phenomena (position of sun, lots of cars parked at front of school).
- Locate a date or event on a calendar (locate when school starts on an official —Gregorian— or local seasonal calendar).
- Make a calendar using markers for times that have personal or local community significance rather than standard times or dates.
- Recognise key times on an analog clock and tell the time of day on digital clocks in hours and minutes.
- Describe how family members' work varies according to the time of day or the time of year.

Using relationships

2.22 No outcome at this level in this sequence.

Level 3 outcomes:

3.18 Selects suitable and uniform things to use as units when measuring and a common unit to compare two things.

3.19 Directly and indirectly compares things, including by counting uniform units of angle, area, capacity and mass and measuring length to the nearest marked graduation.

3.20 Makes sensible numerical estimates using units that can be seen or handled and uses words such as 'between' to describe estimates.

3.21 Measures time and duration of time, reading clocks, calendars and straightforward timetables.

3.22 No outcome at this level.

LEVEL 2 Chance and Data

Level 1 outcomes:

1.23 Shows some recognition of the element of chance in familiar daily activities.

1.24 With guidance, poses questions about collected objects and information.

1.25 Participates in classifying and sequencing objects or pictures.

1.26 & 1.27 Displays objects and pictures and describes data in words and numbers.

Understanding, estimating and measuring chance variation

At level 2, a student:

2.23 Distinguishes possible from impossible events, and describes familiar and easily understood events as more likely or less likely to happen.

Evident when students, for example:

- Respond appropriately to and use 'possible' and 'impossible' for describing familiar events and actions.
- Identify possible and impossible results of a simple and familiar action by thinking about the situation ('You might get a red or a green or a pink jelly bean because they are the colours we put in the bag, but you couldn't get black because we didn't put any in').
- Identify possible results of an action or event by collecting data ('I tried it out and found the hoop could go over no pegs, one peg or two pegs, but it couldn't fit over three because they are too far apart').
- Identify possible outcomes for daily events ('After Daddy picks me up from school, we go to the shops, go straight home, or go and visit a friend').
- Distinguish impossible from unlikely events ('We never go to the park after school, but it isn't impossible, it could happen').
- Describe familiar events as more or less likely to happen ('After school, we are more likely to go ...').

Collecting data

At level 2, a student:

2.24 Contributes questions in group activities and realises some can be answered by collecting data in the form of objects or information.

Evident when students, for example:

- Suggest questions of interest to themselves about their class (their favourite foods, when they get up, what they do on Saturdays, where they were born, what language(s) they speak at home).
- Pose questions suggested by collected data when prompted (having gone on a 'shape walk', they might ask what shapes occur most often in built things; finding there were mostly rectangles in the school building, they ask whether it is so for other buildings).
- Make predictions related to familiar things (think that dogs are the most common animals kept, think sixes are hard to get when you throw a die).
- In answer to the question, 'How could we find out', suggest data collection (objects or information) and offer suggestions about what data to collect (suggest tossing a die to test their belief that sixes are hard to get).
- Collect data in the form of objects, pictures or statements to answer their questions or test their conjectures (ask class members what pets they have in order to test their prediction that the most common type will be dogs).

Organising data

At level 2, a student:

2.25 Contributes to deciding how to classify and sequence data, applying unambiguous and familiar criteria consistently.

Evident when students, for example:

- Offer suggestions about how to classify or sequence objects or information (for a project on animals in their neighbourhood, suggest ways to classify the animals).
- Place measurement data in sensible sequences (cut paper strips to fit around their heads, write names on the tapes, and make a bar —column— graph by lining up the bottoms of strips).
- Apply unambiguous and familiar criteria to classify and sequence data consistently (work in pairs to follow a plan to order the children in their age-group from oldest to youngest, placing cards with name and birthdate written on them in months by birth and then by date in each month).
- With assistance, organise outcomes from simple experiments (having tossed 12 counters, red one side and blue on the other, and listed each outcome as a red/blue pair, follow a suggestion to produce an ordered list of possible red/blue pairs and record how many times each pair appeared).
- Organise their own data by listing items in categories they have created (make up their own food headings and under each list who chose it for lunch).

Displaying and summarising data

At level 2, a student:

2.26 Displays and summarises data based on one-to-one correspondence between data and representation.

Evident when students, for example:

- Experiment with ways of efficiently counting how many there are in each of several categories (use coloured buttons to represent children's snack preferences and arrange buttons in one-to-one correspondence, or draw pictures to represent each snack and underneath record names of children preferring it).
- Compare heights (or lengths) of the columns in a block graph to place categories in order (show more children have a digital watch than one with hands, and the fewest have no watch).
- Make block graphs using real data (children line up by month they were born in, beans used to cover various shapes are lined up).
- Make block graphs using one-to-one correspondence between real data and a representation (one picture for each child of a particular age; a red square for each time red appeared on the spinner).
- Understand the need for a baseline and space blocks regularly (in provided grids) to allow comparisons to be made.

Interpreting data

At level 2, a student:

2.27 Describes, orally and in writing, what own and classmates' displays of data show.

Evident when students, for example:

- Read and compare frequencies from lists and simple one-way tables ('This shows that 4 people like apples and 6 people like bananas').
- Interpret block graphs produced by others (say, 'Their graph shows that most children they asked like tacos better than burritos').
- Describe how their graph shows the results of their data collection ('We spun the spinner 10 times. These three squares on our graph show the three times we got yellow, these [pointing] show we got red six times and green once. Blue didn't come up.').
- Write a few sentences to describe the results of their data collection ('In our 20 tosses we got more heads than tails', 'We found that pizza was the most liked food').
- Comment on information in displays of data produced by themselves and others ('By looking at all the graphs, we can see that some groups got more tails but others got more heads').

Level 3 outcomes:

3.23 Distinguishes certain from uncertain things, describes familiar easily understood events as having equal chance of happening or being more or less likely.

3.24 Contributes to discussions to clarify what data would help answer particular questions or test predictions, and takes care in collecting data.

3.25 Classifies, sequences and tabulates data to help answer particular questions and varies the classification to answer different questions.

3.26 Displays and summarises data using frequencies, measurements and many-to-one correspondence between data and representation.

3.27 Reads and describes information in simple tables, diagrams, pictographs and bar graphs.

LEVEL THREE

LEVEL 3 Statement

Students at level three pay attention to shape and the placement of parts when they make or draw things. Although their drawings only show the visible parts, they can select appropriate nets to match particular solids. They can tell the difference between various solids and use common spatial language to describe features of things in the world around them. They recognise repeating patterns and use repetition to create tiling patterns. They give clear directions of the location and path of things and pay attention to order when reading and making simple maps.

Students can also count, order, estimate and describe using whole numbers, fractions and decimals. They can recognise patterns based on addition and subtraction of whole numbers and use rules to extend the patterns. They remember the basic addition facts and can decide whether to add, subtract, multiply or divide when dealing with everyday problems. They devise strategies for mentally adding and subtracting numbers beyond the basic facts, and can estimate sums and differences and show the relationship between multiplication and division. They use methods that they understand to add and subtract whole numbers and money, and to multiply and divide whole numbers by whole numbers to 10. They also use a calculator efficiently to operate on whole numbers and money.

At this level, students recognise the need for a common unit to compare two things and are able to choose an appropriate unit for the items to be measured. They measure length, capacity, angle, area and mass. They make sensible measurement estimates of things they can see and handle. They can read clocks, calendars and simple timetables, and they can distinguish between actual time and duration of time.

Students draw on their experience to describe familiar things as more or less certain and provide relevant reasons, understanding what it means for two things to be equally likely and using simple examples to explain this. They participate in discussions to decide the kind of data needed for an investigation and decide how best to organise it for presentation. They are able to use tables, diagrams or graphs and to interpret and discuss their own and others' representations.

Students at this level use a variety of strategies for solving problems and confidently carry out necessary checks. They pose questions that might arise in the process of problem-solving and attempt to answer them. They have some knowledge of the mathematics of other cultures and contribute to discussions on the mathematical ways that people have done or thought about things.

WORKING MATHEMATICALLY

SPACE

NUMBER

The description for this strand extends over 4 pages

MEASUREMENT

CHANCE AND DATA

ALGEBRA

LEVEL 3
Table of outcomes

3.1 Poses mathematical questions prompted by similar or related questions or by specific stimulus materials.	3.2 Makes and tests conjectures including by responding to questions of the kind 'what would happen if...'.	3.3 Uses problem-solving strategies that include those based on selecting key information and representing it in models, diagrams and lists.	3.4 Uses a variety of ways when prompted to check working and choice of method.	3.5 Integrates terms and notations from space, number, measurement and chance and data into comparisons and descriptions of things.	3.6 Describes some of the mathematics of own and other cultures, past and present.
See page 52	See page 52	See page 52	See page 53	See page 53	See page 53

3.7a Pays attention to the shape and placement of parts when matching, making and copying things, including matching nets with 3D shapes. 3.7b Matches actual models and conventional drawings of them and attends to what can be seen when making drawings.	3.8 Visualises, follows and gives descriptions of locations and paths and attends to order and proximity in reading and making maps.	3.9 Interprets common spatial language and uses it to describe and compare features of things.	3.10 Recognises and uses repetitions and movements of the same shape embedded within arrangements and patterns.
See page 54	See page 54	See page 55	See page 55

3.11 Counts, orders, estimates and describes with whole numbers, common fractions and decimal fractions for money or measurements (two places).	3.12 Identifies, continues and invents whole number patterns involving the four operations, including where successive terms in a sequence can be linked by an addition or subtraction strategy.	3.13 Uses understanding of whole and fractional numbers, relationships and operations to construct and complete simple statements of equality.	3.14 Makes a suitable choice of operation for situations involving whole numbers, amounts of money and familiar measurements.	3.15 Estimates and calculates mentally, including adding (sum to 100) and subtracting two-digit numbers and multiplying numbers to 10.	3.16 Uses understood written methods to add and subtract any whole numbers and amounts of money and to multiply and divide whole numbers by whole numbers to 10.	3.17 Uses a calculator for operating on whole numbers, amounts of money and measurements.
See page 56	See page 56	See page 57	See page 57	See page 58	See page 58	See page 59

3.18 Selects suitable and uniform things to use as units when measuring and a common unit to compare two things.	3.19 Directly and indirectly compares things, including by counting uniform units of angle, area, capacity and mass and measuring length to the nearest marked graduation.	3.20 Makes sensible numerical estimates using units that can be seen or handled and uses words such as 'between' to describe estimates.	3.21 Measures time and duration of time, reading clocks, calendars and straightforward timetables.	3.22 No outcome at this level
See page 60	See page 60	See page 61	See page 61	

3.23 Distinguishes certain from uncertain things, describes familiar easily understood events as having equal chance of happening or being more or less likely.	3.24 Contributes to discussions to clarify what data would help answer particular questions or test predictions, and takes care in collecting data.	3.25 Classifies, sequences and tabulates data to help answer particular questions and varies the classification to answer different questions.	3.26 Displays and summarises data using frequencies, measurements and many-to-one correspondence between data and representation.	3.27 Reads and describes information in simple tables, diagrams, pictographs and bar graphs.
See page 62	See page 62	See page 63	See page 63	See page 63

3.28 – 3.30 No outcomes at this level

LEVEL 3 Working Mathematically

Level 2 outcomes:

No outcomes at this level

Level 1 outcomes:

1.1 & 1.2 With guidance, poses questions that can in part be answered by classifying, matching, ordering or counting.

1.3 Answers mathematical questions by acting out a story, showing it with objects or pictures, or by trial and error.

1.4 Begins to exhibit some self-correcting behaviour in mathematical activities.

1.5 Talks about mathematical ideas in natural language.

1.6 Has some understanding of how mathematics is used in modifying social behaviour in classrooms and at home.

Investigating

At level 3, a student:

3.1 Poses mathematical questions prompted by similar or related questions or by specific stimulus materials.

Evident when students, for example:

- Pose a question prompted by a number sentence (write a story or question leading to 16 x 7 = ?).
- Generate sets of questions they or peers can answer from data in a table, graph, schedule, advertisement or menu.
- Generate mathematical questions for themselves and others to investigate, stimulated by a familiar context (a photograph of people at a country football game or a set of photographs of different hub caps from car wheels).
- Produce variations on problems explored (investigate 'How tall was the giant?' from Jack and the Beanstalk, then suggest similar questions such as 'How much did the golden egg weigh?').
- Generate more problems for solving using a strategy just developed (having worked out their own method for finding how many days in 8 years, students write down other questions this method would be useful for answering).
- Contribute questions in a brainstorming situation to help deal with a practical task, such as when planning a doughnut-eating contest, suggest questions about the source of doughnuts (Who supplies? At what prices?), their placement (How tall will people be? How low should they be hung? How far apart? Where? How?), and charges.

Conjecturing

At level 3, a student:

3.2 Makes and tests conjectures including by responding to questions of the kind 'what would happen if ...?'.

Evident when students, for example:

- Offer suggestions in response to 'What would happen if ...' questions (What would happen if we covered the leaf with smaller beans? If we'd used a different shaped rectangle for our tiles? If we'd used triangles instead of rectangles?).
- Make conjectures about operations on numbers (say, 'If I start with 48 – 30 and then keep increasing each number by the same amount, the answer will stay the same').
- Visualise the effect of folding and unfolding to make predictions and test them (having checked whether a particular pentomino can be folded to make an open box, visualise what would happen if a square was in a different position).
- Collect data to assist in making and testing a conjecture (in response to 'What happens if you add two odd numbers', try various different odd numbers and conjecture that two odd numbers always give a sum that is even).
- Try to convince themselves and others of the correctness of their conjectures by drawing on their mathematical knowledge (having shown how any odd number can be represented by 'pairs of counters and one over', argue that when adding two odd numbers the 'one over' in one number pairs with the one over in the other).
- Use sequences of number 'facts' to generate rules that assist in mental arithmetic (use the pattern in 16 – 9, 26 – 9, 36 – 9 to predict what 86 – 9 will be).

Using problem-solving strategies

At level 3, a student:

3.3 Uses problem-solving strategies that include those based on selecting key information and representing it in models, diagrams and lists.

Evident when students, for example:

- Attempt to understand essential features of a problem by asking such questions as 'What do we know', 'What do we want to know?' 'What am I trying to find out?' 'Can I say it in my own words?'
- Decide what information in a problem needs to be represented (in a problem about the total number of wheels on bicycles and cars, all that needs to be represented is the wheels, or number of wheels, on each object).
- Use guess and check as a strategy, recording guesses (given a description of a shape, guess triangle and test against the description; if it isn't right, record it, guess another shape and test).
- Represent a problem with concrete materials and manipulate the materials to find a solution (using four cubes, investigate and record the number of possible arrangements).
- Make lists or tables of data collected to help solve a problem (produce a list of rectangles that can be made with 36 square tiles and record the area of each rectangle).
- Use arrays and tree diagrams to make organised counts (find how many different garments can be made given a choice of 5 colours, 6 styles and 3 sizes).

Applying and verifying

At level 3, a student:

3.4 Uses a variety of ways when prompted to check working and choice of method .

Evident when students, for example:

- Respond to questions such as 'Is your working correct'? by checking each stage of the work (redo the calculation, check that each classified item is placed in the right category, reorder the objects and events and compare with first attempt, start again and flip the shape the same way each time).
- Respond to questions such as 'Is there another way you could check your answer?' by doing it in a different way (do 17 + 7 + 13 in a different order, check that 327 – 232 is 95 by adding 232 and 95, use a different unit to compare the lengths).
- Check their computation by approximation (327 – 232 will be a bit less than 327 – 227, which is a bit less than 100).
- Check their answers against their estimates and reconsider both their methods and their calculations if results seem unreasonable (say, 'This means that box is the biggest but it doesn't look that way to me, I'd better check ...').
- Check for transcription errors (that data are correctly recorded in their table).

Using mathematical language

At level 3, a student:

3.5 Integrates terms and notations from space, number, measurement and chance and data into comparisons and descriptions of things.

Evident when students, for example:

- Read and write numerical statements involving the four operations (18 – (24 – 7) = 1).
- Relate alternative everyday language expressions to one arithmetic expression (45 ÷ 5 = ? may be read 'What is 45 divided by 5?' or 'How many lots of 5 in 45?').
- Integrate conventional names of shapes and parts of shapes into their descriptions of things.
- Integrate measurement language (attributes, comparative language and units) into sentences when describing and comparing things.
- Identify and talk about words that have a mathematical meaning which is different from everyday usage (table, odd).
- Use mathematical language in non-mathematical situations (use the mathematical language of position when describing their map in social studies, use the informal language of transformation in describing their patterns in art).

Working in context

At level 3, a student:

3.6 Describes some of the mathematics of own and other cultures, past and present.

Evident when students, for example:

- Make and explain devices that have a mathematical basis and which people have used in the past for practical tasks (a candle clock, pendulum or sundial for measuring time; tally sticks, abacus, systems of knots for counting and recording numbers).
- Demonstrate counting and numeration systems from cultures past and present (Aboriginal and Torres Strait Islander counting systems such as the tally system based on five taught to drovers and stockmen by Aboriginal people).
- Describe mathematical features of some games and puzzles popular in different communities, past or present (tangrams and magic squares from China).
- Find out about and report clearly to peers on some mathematics used at work or at home by a family or community member.
- Use and explain some procedures for carrying out mathematical tasks which are alternatives to their own methods (demonstrate and compare a standard algorithm for subtraction from a modern Japanese school book, an Australian one and one from the Netherlands).
- Identify and reproduce spatial features in spiritual, aesthetic and leisure objects from their own and others' cultures or cultures of origin (the symmetries, circles and other shapes in the signs of major world religions, in the symbolic art of some Aboriginal and Torres Strait Islander clans, in the design of playing grounds for sporting games).

Level 4 outcomes:

No outcomes at this level

Level 5 outcomes:

5.1 Begins and extends tasks by asking some mathematical questions, including 'What would happen if...?'.

5.2 Understands a conjecture as a guess with reasons and draws on mathematical knowledge to give reasons for conjectures before testing them.

5.3 Uses problem-solving strategies, including those based on selecting and organising key information and being systematic.

5.4 Checks that answers fit specifications and make sense in the original situation.

5.5 Uses mathematical terms and notations with some care to describe objects and relationships and report conclusions with clarity.

5.6 Explains some ways mathematics is used, or has been used in the past, to represent, describe and explain our world.

LEVEL 3 Space

Level 2 outcomes:

2.7a Fulfils simple spatial criteria when making things from verbal or visual descriptions.

2.7b Pays attention to the shape and placement of component parts when interpreting and making drawings.

2.8 Follows and gives descriptions of locations and paths and pays attention to order in reading and making informational maps.

2.9 Interprets common spatial language and, when prompted, uses it to describe the shape of things.

2.10 Generates patterns and follows rules based on the simple repetition and movement of things.

Using spatial ideas, tools and techniques to interpret, draw and make

At level 3, a student:

3.7a Pays attention to the shape and placement of parts when matching, making and copying things, including matching nets with 3D shapes.

Evident when students, for example:

- Select and fit together suitable pieces from construction materials to make models they can see and handle.
- Make polyhedra in solid (with clay), hollow (with provided nets) and skeleton (with straws) forms.
- Discuss the features of 3D shapes emphasised by and best represented in solid, hollow and skeleton forms.
- Inspect a prism or pyramid, put it aside and then select from a collection of figures (2D shapes) a set matching the faces of a polyhedron, making sketches or notes if it helps.
- Inspect a polygon, prism or pyramid and tell a friend what sticks and joiners to collect to make the shape (ask for six equal lengths of straw and six joiners to make a copy of a regular hexagon).
- Match clearly different right prisms and pyramids with nets by considering the number, shape and placement of the faces.
- Predict which pentominoes can be used as nets for an open box and test predictions by folding.

3.7b Matches actual models and conventional drawings of them and attends to what can be seen when making drawings.

Evident when students, for example:

- Talk about what they can and cannot see of an object from different positions and try to draw what they see rather than what they know to be there.
- Match a representation with the point of view from which it was made (move around the schoolyard until what they see matches a photograph).
- Arrange objects to match realistic drawings, paintings and photographs and provide some form of pictorial information to enable others to reproduce arrangements (position people in the yard roughly as shown in a photograph or sketch).
- Sequence photographs or drawings to match a physical reality (as they walk around an arrangement or object).
- Match standard geometric models with realistic drawings and conventional diagrams (match a prism with a conventional drawing of it).
- Imagine and draw different cross-sections of simple 3D shapes and then check and improve the drawings by observing the cross-section (slicing carrots at different angles).

Visualising, analysing and representing arrangements and locations

At level 3, a student:

3.8 Visualises, follows and gives descriptions of locations and paths and attends to order and proximity in reading and making maps.

Evident when students, for example:

- Recognise a map as a bird's-eye view and attempt to provide this view of familiar locations such as their classroom.
- Attend to the order and proximity of things in drawing maps, giving directions or describing the main features of a familiar path (correctly place the bus-stop between the park and shop but closer to the shop, saying 'As soon as you get to the tree look to the right...').
- Interpret order and proximity from maps ('This shows that her place is just after Beryl's, but then it is a lot further to our place').
- Use directional language (use north, south, east and west, right angle, quarter turn when describing their map in social studies).
- Visualise and test the effect of following a rule involving a simple sequence of movements (what would happen if we went 'two steps forward, turn right, two steps, turn right, two steps, turn right'... or 'one step forward, turn right, two steps, turn right, three steps, turn right...').

Visualising, analysing and representing shapes

At level 3, a student:

3.9 Interprets common spatial language and uses it to describe and compare features of things.

Evident when students, for example:

- Integrate conventional names of shapes and parts of shapes into their descriptions of things (side, face, edge, vertex, base, surface, curved, triangular, circular).
- Identify the spatial features of things and link shape to functions such as stability, strength, storage (compare a milk bottle and a milk carton).
- Identify prisms and pyramids (including cones and cylinders) in their environment ('This tent is very like a prism with triangular ends but the other one is a pyramid on a square base').
- Describe and compare the spatial features of various mathematical objects they can see and handle, including by the number of edges and vertices and the number, shape and position of faces.
- Visualise and describe cross-sections of familiar objects ('We thought all the carrot slices would be circles but they weren't. Some were like long circles and we found they are called ellipses.').
- Given a description of its spatial features, select an object from a collection, ('I have two flat faces and one curved face. I also have two edges. What am I?').

Visualising, analysing and representing movements and transformations

At level 3, a student:

3.10 Recognises and uses repetitions and movements of the same shape embedded within arrangements and patterns.

Evident when students, for example:

- Find by observation, tracing and cutting repetitions of figures and objects in decorative patterns (tiling, fabrics), objects (beehives, blocks of flats, stacked tetrapaks), and formations (dance routines, marching groups).
- Predict whether a part of a visual design will fit exactly over another, test and describe conclusions ('We thought the border was made from two different shapes, but we traced them and found they were the same shape flipped over').
- Use multiple copies of shapes to create patterns that use reflection, rotation and translation —not necessarily the terminology— systematically (plan and make a border).
- Copy a pattern based on systematic movement of shapes and informally describe the 'system' that produced it.
- Provided with multiple copies of a figure, decide if it will fit together to cover a surface without gaps or overlaps—tessellate.
- Explain why they think particular shapes will not tessellate no matter how long they experiment ('You can't make the corners fit together at the same point and if you put the corner of one piece against the side of another, nothing will fit in the gap').

Level 4 outcomes:

4.7a Shows care in the shape, size and placement of parts when they match, make and copy things, including making nets of 3D shapes by drawing around their faces.

4.7b Interprets and makes drawings of 3D shapes, using basic conventions for representing 3D space in 2D.

4.8 Visualises, follows and describes locations and paths and reads and makes maps and plans, using distance, direction, coordinates and scales.

4.9 Selects, describes and compares figures and objects on the basis of spatial features, using conventional geometric criteria.

4.10 Recognises and uses rotations, reflections and translations to relate the features of arrangements and patterns.

LEVEL 3 Number

Level 2 outcomes:

2.11 Counts, orders, estimates and describes with whole numbers within everyday experience and with fractions expressed in words.

2.12 Identifies, continues and invents whole number patterns, including where successive terms in a sequence can be linked by addition or subtraction of a constant amount.

2.13 Uses understanding of whole numbers and addition and subtraction relationships to construct and complete simple statements of equality.

2.14a Represents problems involving the four basic operations on whole numbers in a variety of ways, using stories, materials, pictures and number sentences.

2.14b Compares prices with money available, calculates the exact and approximate cost of several items and makes change.

2.15 Estimates and calculates mentally, including adding and subtracting numbers to 10 and making extensions based on place value.

2.16 Uses a variety of strategies, including regrouping, to assist in adding and subtracting whole numbers when unable to complete mentally.

2.17 Uses a calculator to represent and explore numbers, place value and operations.

Count and order

At level 3, a student:

3.11 Counts, orders, estimates and describes with whole numbers, common fractions and decimal fractions for money or measurements (two places).

Evident when students, for example:

- Use multiples — equivalent groups — to count and compare collections (take three at a time, saying 3,6,9,12, and two, 12, 14).
- Use place value to read, write, say and interpret whole numbers, oral or written (order capital cities by their populations).
- Use place value to assist in estimation (round numbers to the nearest 10 or 100, or rounding up or down).
- Use decimal notation to two places to represent quantities and money (use 1.25m for 1m 25 cm, $3.05 for three 1 dollar coins and one 5 cent coin, 1.75 kg for 1750 g).
- Regroup money to fewest possible number of notes or coins (exchange eleven $5 notes, seventeen $2 coins and eight $1 coins for a $50 note, two $20 notes and $5 note and a $2 coin).
- Use materials and diagrams to represent fractional amounts where the whole may be an object, quantity or collection (fold tape into 5 equal parts and shade 3 parts to show $\frac{3}{5}$).
- Express generalisations about fractional numbers symbolically (write 'We found one quarter of a thing was always the same as two eighths of it'; replace with '1 quarter = 2 eighths' and $\frac{1}{4} = \frac{2}{8}$).
- Count in common fractional amounts (two and one-third, two and two-thirds, three...).

Number patterns

At level 3, a student:

3.12 Identifies, continues and invents whole number patterns involving the four operations, including where successive terms in a sequence can be linked by an addition or subtraction strategy.

Evident when students, for example:

- Build sequences of simple shapes such as triangles, squares, L or T shapes which increase in size systematically and write the equivalent number pattern (build 'growing' triangles from counters leading to the triangular numbers 1, 3, 6, 10...).
- Describe a rule that could be used to generate a sequence involving an addition or subtraction strategy which may involve more than adding or subtracting a constant amount (start with 2 and add 1, then add 2, then add 3...).
- Describe a rule that could be used to generate a multiplying or repeated addition sequence (start with 5 and double each time).
- Continue or fill in number sequences based on an addition or subtraction strategy involving more than adding or subtracting a constant amount, (2, 5, 9, 14, ... , 27, ...).
- Follow a rule to generate a number sequence based on an addition or subtraction strategy ('Start with 1, 2, then each time add the two previous numbers').
- Identify and describe patterns in multiplication tables (in the nine times table the digits always add to nine, the units go from 9 down to 0...).
- Use patterns in digits to make predictions ('If you were counting by fours and started with 23, would you see 87 on the calculator?').

Equations

At level 3, a student:

3.13 Uses understanding of whole and fractional numbers, relationships and operations to construct and complete simple statements of equality.

Evident when students, for example:

- Use addition/subtraction and multiplication/division relationships to solve missing number problems (solve several such as [ ] + 13 = 20 either mentally or with a calculator, find a method that always works, and use it to solve [ ] + 238 = 451 on a calculator).
- Complete numerical statements involving brackets where only one number is 'missing' (such as (19 + 6) x _ = 50 or (19 + _) x 2 = 50).
- Use specified numbers to generate equations that satisfy numerical constraints (use 3, 4, 6 and 7, put one number in place of each question mark to make a true sentence: ? + ? – ? = ?).
- Use patterns in sequences of number equations to generate new equations (fill in the missing numbers in 18 – 14 = , 28 – 14 = , 38 – 14 = , ?? – 14 = 34, 58 – ?? = 44, ?? – ?? = 54, ?? – ?? = and use the pattern to decide 298 – 14=_).
- Use the relationship between multiplication and division to solve missing number problems (to find the missing number in: _ x 7 = 497 rewrite it as: 497 ÷ 7 = _).
- State fractional equivalents in words and symbols ('We found one quarter of the chocolate was the same as two eighths of it, we found the same for the water, and for the paper clips. We think it will be so no matter what we use so 1 quarter = 2 eighths, and ¼ = ²⁄₈).').

Applying numbers

At level 3, a student:

3.14 Makes a suitable choice of operation for situations involving whole numbers, amounts of money and familiar measurements.

Evident when students, for example:

- Select an appropriate multiplication to deal with repeated addition ('The cat eats half a 500 g can of cat food each day. If I buy it in bulk, how much per week will I need?') and cross-product situations ('How many different shoes can be made with 4 types of sneakers and 5 colours of laces?').
- Select an appropriate division to deal with sharing and grouping (including measurement) where the divisor is a whole number and smaller than the dividend (to find how much each class will need to raise if the school needs to raise $1000, to find how many portions of —— is in —— ?).
- Restate multiplication and division problems symbolically (say, 'Each side of the banner is 1.35 m so to find the distance around it I need to find 1.35 x 4').
- Suggest a story question that relates to a given number sentence (write a story or question leading to 35 x 15 = ?).
- Match word problems with particular calculations (given four problems all using 19 and 7 — and possibly other numbers — select those which can be solved by 19 x 7 = _).

LEVEL 3 Number

Mental computation

At level 3, a student:

3.15 Estimates and calculates mentally, including adding (sum to 100) and subtracting two-digit numbers and multiplying numbers to 10.

Evident when students, for example:

- Remember basic addition facts and some multiplication facts and calculate mentally basic multiplication facts they don't recall (3 eights is 24 so 6 eights would be double that, so 6 x 8 = 48).
- Use the relationship between multiplication and division to state division facts (5 eights is 40 so 8 goes into 40 five times).
- Use place value to make mental extensions of multiplication facts (3 lots of 5 is 15, 3 lots of 50 is 3 lots of five tens, 15 tens, 150).
- Decompose two-digit numbers to assist in adding and subtracting them mentally ('To take off 23, I took off the 20 and then I took off 3 more' or '46 and 23 is 46, 56, 66 plus 3 which is 69').
- Decompose one-digit numbers to assist in multiplying them (7 nines is 7 tens take 7 ones, which is 70 take 7 or 63).
- Add a written list of one-digit numbers in an order that makes it easier ('To find 6 + 7 + 4 + 5, I found it is easiest to add 6 + 4 to make 10, and then add 7 and 5').
- Estimate sums and products by rounding to single digit numbers ('The mugs cost $4.95, so $30 will be enough for six').

Written computation

At level 3, a student:

3.16 Uses understood written methods to add and subtract any whole numbers and amounts of money and to multiply and divide whole numbers by whole numbers to 10.

Evident when students, for example:

- Add columns of whole numbers and subtract one number from another (add 264, 121, 439 by adding hundreds, tens and units separately and writing 700 + 110 + 14 = 824; write 457 – 361 = 157 – 61 = 100 – 4 = 96, or use a conventional algorithm).
- Give a reasonable explanation of why their written method for adding or subtracting numbers works by referring to place value.
- Multiply by single digit numbers (for 27 x 4 write 27, 54, 108 or (20 + 7) x 4 = 80 + 28 = 108, or use a conventional algorithm).
- Divide a whole number by a one-digit number, expressing the results with remainders or fractions (to find how many weeks pass in 127 days think of 127 as 70 + 57 and record as 10 + 8 with remainder 1, which is 18r1, or use a conventional algorithm).
- Explain why their multiplication method works (represent 27 in MAB as two longs and 7 units and show that you can find 4 lots of the longs and 4 lots of the units and combine them).
- Compare paper-and-pencil methods for ease and efficiency ('For 47 + 36 it's quicker to write down 70 and 13 to get 83. But for lots of numbers, lining them up is quicker and stops mistakes.').

Calculators

At level 3, a student:

3.17 Uses a calculator for operating on whole numbers, amounts of money and measurements.

Evident when students, for example:

- Enter subtractions in the correct order to answer their questions and interpret answers (for how much more they need to raise if they have $125 and need $180, enter [1][8][0][−][1][2][5][=]).
- Use a calculator to investigate patterns in related computations (find 3 x 3 = , 33 x 33 =, 333 x 333 =, predict the next answer and check).
- Enter and read amounts of money and measurements on a calculator, rounding calculator displays to the nearest cent or unit, and recognising, for example, 1.5 as $1.50.
- Explain that the multiplication key of a calculator is more efficient than repeated addition for finding the cost of several items of the same price and explain its use to a peer.
- Mentally estimate the results of a calculation to check the reasonableness of results gained with a calculator.

Level 4 outcomes:

4.11 Counts, orders estimates and describes with common and decimal fractions.

4.12 Identifies, continues and invents whole and fractional number patterns, including those where successive terms in a sequence can be linked by a multiplication or division strategy.

4.13 Uses understanding of numbers and number relationships to construct and complete statements of equality, including those where more than one solution exists.

4.14 Makes a suitable choice of operation involving whole and fractional numbers (whole number multipliers and divisors), including those where more than one operation is needed.

4.15 Estimates and calculates mentally, including adding and subtracting most two-digit numbers and multiplying and dividing multiples of 10 by one-digit numbers.

4.16 Uses understood written methods to add, subtract, multiply and divide whole numbers, money and measures (two places, whole number multipliers and divisors to 10).

4.17 Uses a calculator efficiently for operating on decimal numbers, including where more than one operation is needed, and interprets displays for division.

LEVEL 3 Measurement

Level 2 outcomes:

2.18 Chooses the appropriate physical attribute when comparing and measuring things and units which relate to that attribute.

2.19 Directly and indirectly compares lengths and capacities, and measures and makes lengths and distances by counting uniform units, including metres and centimetres.

2.20 Estimates the order of things by length, area, mass and capacity and makes numerical estimates of length using a unit that can be seen or handled.

2.21 Locates and sequences events in time, beginning to read familiar clocks and calendars.

2.22 No outcome at this level.

Choosing units

At level 3, a student:

3.18 Selects suitable and uniform things to use as units when measuring and a common unit to compare two things.

Evident when students, for example:

- Understand that units are needed when direct comparison is not possible or we wish to know, that is, quantify, 'How big' or 'How much bigger'.
- Choose same size objects to use as a repeated unit for measuring (select uniformly sized over irregularly sized pebbles to balance objects to measure mass in pebbles).
- Refer to gaps and overlaps in explaining differences in the number of units taken to cover an object ('We used the same tiles but he pushed his close together and I left mine spread out. If we cover the book without any gaps we should both get the same answer.').
- Choose shapes that can cover a region with no gaps or overlaps (that is tile or tessellate) to use as units of area.
- Explain differences in the number of units different students took to cover an object by referring to the different sizes of the units chosen ('The jar took 9 of my scoopfuls but 12 of her eggcups. That's because my scoops held more than her eggcup.').
- Understand the necessity of selecting the same unit when comparing two things (say, 'We used my strides for the room and it was five strides. The carpet is just under five of Kaye's strides but it still may not fit if Kaye's strides are bigger.').

Measuring

At level 3, a student:

3.19 Directly and indirectly compares things, including by counting uniform units of angle, area, capacity and mass and measuring length to the nearest marked graduation.

Evident when students, for example:

- Use indirect methods to order objects when direct comparison is not possible (to compare the area of differently shaped sheets of paper, cut and rearrange the pieces of one and fit over the other).
- Order regions by direct comparison of area (superimposition) and by using a unit (count same size tiles and part-tiles to measure and order the states of Australia by size on map).
- Identify angles in turns or rotations (movement of hands of a clock, opening of a book) and by the joins of lines (intersections of walls).
- Order angles by direct comparison of the amount of turn and by using units such as a quarter turn or an angle of their own making.
- Use numerical measurements of objects to order the objects (order three or more objects of a similar visual size by mass, by counting and recording how many marbles balance each object).
- Use the number of uniform bricks or cubes as a measure of volume (compare the size of their construction with their friend's by counting the number of cubes used).
- Measure length to the nearest marked graduation (read a tape measure where the centimetres are marked but only the metres and decimetres are labelled).

Estimating

At level 3, a student:

3.20 Makes sensible numerical estimates using units that can be seen or handled and uses words such as 'between' to describe estimates.

Evident when students, for example:

- Make sensible numerical estimates based on provided units (how many gum nuts like this one will fill the can).
- Make informal statements about how confident they are about their estimates ('I am sure the wall is between 3 and 6 metres wide, pretty sure it is between 4 and 5, and think it is closer to 4').
- Express estimates and measures with 'between' statements ('I think the jug will hold between 5 and 6 cups').
- With a physical model of the unit available for comparison, estimate which regions they can see have an area of about or less than or more than a square metre, which containers have capacity of about or less than or more than a litre, which objects have a mass of about or less than or more than a kilogram.
- Realise when a numerical estimate isn't needed because a judgement by eye will do (decide by looking that the cupboard will fit the corner, but can't tell if the bookcase will fit and therefore measure it).
- Consciously use the result of measuring with a unit to try to improve estimates with the unit (how many marbles —the unit— balance various stones?).
- Use body parts and movements as a unit to help estimate length (know which body parts and movements are about 1 cm, 10 cm and 1 m long).

Time

At level 3, a student:

3.21 Measures time and duration of time, reading clocks, calendars and straightforward timetables.

Evident when students, for example:

- Describe different ways of thinking about day and when the day begins (officially the day begins at midnight, but we often think of it as beginning at sunrise and Anindilyakwa people actually regard the day as beginning at sunrise).
- Estimate time of day, week or year using clues such as shadows, weather, clothing, fullness of car park, shop signs, plant behaviour.
- Tell the time on digital and analog clocks.
- Read and make simple schedules (for the school sports day, the TV guide).
- Ask and answer questions about how long things took or will take and measure time intervals using natural units of time (sunsets, new moons), artificial non-standard units (rhythmic body movements), timers (metronome) or standard units (stopwatch).
- Classify events into those that take more than or less than or about one hour or one half hour or five minutes.
- Work out the time a certain number of minutes or hours ago or ahead (to set the alarm on a watch for one half hour hence).

Using relationships

3.22 No outcome at this level in this sequence.

Level 4 outcomes

4.18 Selects appropriate attributes and units of a sensible size for the descriptions and comparisons to be made.

4.19 Measures and makes things, using conventional units and measuring equipment for length, mass, capacity and angle and reading scales to the nearest marked graduation.

4.20 Uses the known size of familiar things to help make and improve estimates, including those with centimetres, metres, kilograms and litres.

4.21 Estimates and measures time and duration of time and prepares feasible timetables.

4.22 Understands and uses relationships involving perimeters of polygons and areas of regions based on squares.

LEVEL 3 Chance and Data

Level 2 outcomes:

2.23 Distinguishes possible from impossible events, and describes familiar and easily understood events as more likely or less likely to happen.

2.24 Contributes questions in group activities and realises some can be answered by collecting data in the form of objects or information.

2.25 Contributes to deciding how to classify and sequence data, applying unambiguous and familiar criteria consistently.

2.26 Displays and summarises data based on one-to-one correspondence between data and representation.

2.27 Describes, orally and in writing, what own and classmates' displays of data show.

Understanding, estimating and measuring chance variation

At level 3, a student:

3.23 Distinguishes certain from uncertain things, describes familiar easily understood events as having equal chance of happening or being more or less likely.

Evident when students, for example:

- Use data to compare events within their experience, describing them as more and less likely (it is more likely to rain in Bathurst in July than in January).
- Describe outcomes as having an equal chance or being equally likely (a head has an equal chance with a tail, a baby is equally likely to be born on any day of the week).
- Order a few easily understood situations from least likely to most likely (tomorrow will be Sunday, my teacher will come to school tomorrow with green hair).
- Justify their choice of more or less likely by referring to experience or information (say, 'I don't think my teacher will come to school tomorrow with green hair — it's almost impossible. But it is even less likely tomorrow will be Sunday because today is Tuesday, so that's impossible.').
- Make informal statements about how one might influence the chance of an event happening (say, 'I am less likely to have an accident if I take the back road because I don't have to cross any busy streets').

Collecting data

At level 3, a student:

3.24 Contributes to discussions to clarify what data would help answer particular questions or test predictions, and takes care in collecting data.

Evident when students, for example:

- Make predictions about familiar people, things, events and environments (which sports gear gets used most, which colour will come up most often on a spinner, which beans sprout fastest, which shape is used most often in packaging).
- Suggest information to collect to answer particular questions (to see if older children find different things scary, they may decide to have a class of older children draw pictures of scary things and compare them with their own).
- Clarify questions to decide what data to collect ('We want to know what are the most popular pets in the class — and we mean best liked, not what most children have').
- Specify how frequencies or measurements are to be made (a bag of marbles counts as one for marbles, not how many are in the bag; arm length is from inside the armpit to the wrist).
- Record frequency data carefully using simple formats based on tallies or organised lists and take care with their measurements (keep a tally of the results of tossing a die 60 times).

Organising data

At level 3, a student:

3.25 Classifies, sequences and tabulates data to help answer particular questions and varies the classification to answer different questions.

Evident when students, for example:

- Suggest a suitable way to classify and sequence data to answer straightforward questions (to describe how children get to school, they suggest sorting their 'coming to school' sketches into groups by means of transport — car, bus, bike or walking).
- Realise that different classifications tell different things and suggest other classifications to answer new questions (compare 'coming to school' pictures sorted by school-goer's company, for example, alone, with another child, with adult, or by form of transport; to answer how many children need to cross the intersection, simplify the transport classification into 'walk' and 'don't walk').
- Improve their descriptions of categories to clarify what the category includes or excludes (when classifying drawings of 'things that scare me' into 'alive' and 'not alive', decide where 'ghosts' belong).
- Collaboratively plan an efficient way to sequence data (to order rocks by mass, group by kilograms and order within each group).
- Use diagrams such as Venn or Carroll diagrams and two-way tables to represent a two-way classification (place name cards in appropriate sections of their diagram or table).

Displaying and summarising data

At level 3, a student:

3.26 Displays and summarises data using frequencies, measurements and many-to-one correspondence between data and representation.

Evident when students, for example:

- Summarise data based on tallying (use a conventional tally method to record the number of times a thumb tack falls on its side or on its top and summarise by counting the tally in its groups).
- Summarise data in diagrams and tables that show frequencies for different categories (category may be type of food, and frequency the number of children choosing that type; names of children recorded in a Venn diagram are replaced by the count of how many in that category).
- Use lengths to represent other measures such as time or mass (mark paper strips with the 24 hours of the day, shade the time between going to bed and getting up, cut shaded parts and make a graph of the times in bed).
- Display data in pictographs where each symbol represents more than one unit, such as one picture for each ten children.
- Display frequency data in bar graphs (vertical and horizontal) where one axis shows the whole numbers (0, 1, 2, 3...).

Interpreting data

At level 3, a student:

3.27 Reads and describes information in simple tables, diagrams, pictographs and bar graphs.

Evident when students, for example:

- Interpret straightforward one- and two-way tables ('This shows that of the 17 berries, 10 were green and edible, 7 were green and not edible...').
- Read frequencies from a bar graph (with each unit on the frequency axis marked) and describe the data ('Their graph shows that of the people asked 11 said they preferred frozen yoghurt').
- Interpret pictographs where each picture represents more than one unit ('This shows that there were a few more than 40 dogs and almost 30 cats).
- Comment upon their predictions in the light of results ('We thought that other children would find the same things scary but that isn't what we found. We now think that what makes you scared might change as you get older', 'We thought that blue smarties would come out least and we were right').

Level 4 outcomes:

4.23 Places events in order from those least to most likely to happen on the basis of numerical and other information about the events.

4.24 Collaborates in deciding how data collection could help investigate situations and problems, frames helpful questions and decides what data to collect.

4.25 Classifies, sequences and tabulates data, with some grouping of data, and chooses methods helpful for answering particular questions.

4.26 Displays frequency and measurement data using simple scales on axes and summarises data with simple fractions, highest, lowest and middle scores, and means.

4.27 Reads and describes information in tables (including some grouping of data), diagrams, line and bar graphs, fractions and means.

LEVEL FOUR

LEVEL 4 Statement

Students at level four know some more of the common conventions for drawing three-dimensional shapes. They use these conventions to draw common shapes, not necessarily with precision, and use them in drawing other objects. They make nets of shapes by tracing around their edges and predict whether or not folding a net will produce a particular three-dimensional shape. They use geometrical language to describe figures and predict whether shapes will tile. They use map coordinates and simple scales to interpret and make maps, and plan and describe routes on a map to fit specifications. They take into account the purpose of a map when selecting features to highlight and to determine whether a scale is needed. They give clear and simple instructions for finding things in the environment or on models.

Students at this level read and make sense of common fractions and order decimals with equal numbers of places. They investigate number patterns involving multiplication and division. They solve simple number puzzles and, when prompted, use relationships between numbers to assist their calculations and estimations. They begin to understand that multiplication is used for more than repeated addition and can connect multiplication with the idea of scale. They make decisions about when to use each of the four operations in a wide range of practical situations. They know the basic addition and multiplication facts and use them to state related subtraction and division facts. They use a variety of strategies for mental calculation involving both exact and approximate answers. They add and subtract numbers having the same number of decimal places and can multiply and divide whole numbers and decimals by single digit numbers. They use a calculator to carry out computational tasks and can interpret remainders when using a calculator for division.

Students at this level are also developing a sense of the size of common standard units. They know the size of common things and use these to help in estimation. They are beginning to understand that their choice of unit should be influenced by the fineness of the comparison or fit to be made rather than just the size of the object to be measured. They can read simple scales and measure accurately to the nearest marked graduation. They have begun to discover relationships between measurements for some common shapes. The difference between time and duration of time is now much clearer to the students, and they make reasonable estimates of time and use familiar events to assist them with estimations.

Students use a range of sources of information to put things in order of least likely to most likely. They experiment with organising data in forms that help answer questions and can summarise data using fractions, means, and lowest, highest and middle scores. They can group data and produce diagrams, tables and bar graphs. They can interpret tables and graphs and investigate a wide range of practical problems not obviously mathematical.

WORKING MATHEMATICALLY

SPACE

NUMBER

The description for this strand extends over 4 pages

MEASUREMENT

CHANCE AND DATA

ALGEBRA

LEVEL 4
Table of outcomes

4.1 – 4.6 No outcomes at this level

4.7a Shows care in the shape, size and placement of parts when they match, make and copy things, including making nets of 3D shapes by drawing around their faces.

4.7b Interprets and makes drawings of 3D shapes, using basic conventions for representing 3D space in 2D.

See page 68

4.8 Visualises, follows and describes locations and paths and reads and makes maps and plans, using distance, direction, coordinates and scales.

See page 68

4.9 Selects, describes and compares figures and objects on the basis of spatial features, using conventional geometrical criteria.

See page 69

4.10 Recognises and uses rotations, reflections and translations to relate the features of arrangements and patterns.

See page 69

4.11 Counts, orders, estimates and describes with common and decimal fractions.

See page 70

4.12 Identifies, continues and invents whole and fractional number patterns, including those where successive terms in a sequence can be linked by a multiplication or division strategy.

See page 70

4.13 Uses understanding of numbers and number relationships to construct and complete statements of equality, including those where more than one solution exists.

See page 71

4.14 Makes a suitable choice of operation involving whole and fractional numbers (whole number multipliers and divisors), including those where more than one operation is needed.

See page 71

4.15 Estimates and calculates mentally, including adding and subtracting most two-digit numbers and multiplying and dividing multiples of 10 by one-digit numbers.

See page 72

4.16 Uses understood written methods to add, subtract, multiply and divide whole numbers, money and measures (two places, whole number multipliers and divisors to 10).

See page 72

4.17 Uses a calculator efficiently for operating on decimal numbers, including where more than one operation is needed, and interprets displays for division.

See page 73

4.18 Selects appropriate attributes and units of a sensible size for the descriptions and comparisons which are to be made.

See page 74

4.19 Measures and makes things, using conventional units and measuring equipment for length, mass, capacity and angle and reading scales to the nearest marked graduation.

See page 74

4.20 Uses the known size of familiar things to help make and improve estimates, including those with centimetres, metres, kilograms and litres.

See page 75

4.21 Estimates and measures time and duration of time and prepares feasible timetables.

See page 75

4.22 Understands and uses relationships involving perimeters of polygons and areas of regions based on squares.

See page 75

4.23 Places events in order from those least to most likely to happen on the basis of numerical and other information about the events.

See page 76

4.24 Collaborates in deciding how data collection could help investigate situations and problems, frames helpful questions and decides what data to collect.

See page 76

4.25 Classifies, sequences and tabulates data, with some grouping of data, and chooses methods helpful for answering particular questions.

See page 77

4.26 Displays frequency and measurement data using simple scales on axes and summarises data with simple fractions, highest, lowest and middle scores, and means.

See page 77

4.27 Reads and describes information in tables (including some grouping of data), diagrams, line and bar graphs, fractions and means.

See page 77

4.28 – 4.30 No outcomes at this level

LEVEL 4 Space

Level 3 outcomes:

3.7a Pays attention to the shape and placement of parts when matching, making and copying things, including matching nets with 3D shapes.

3.7b Matches actual models and conventional drawings of them and attends to what can be seen when making drawings.

3.8 Visualises, follows and gives descriptions of locations and paths and attends to order and proximity in reading and making maps.

3.9 Interprets common spatial language and uses it to describe and compare features of things.

3.10 Recognises and uses repetitions and movements of the same shape embedded within arrangements and patterns.

Using spatial ideas, tools and techniques to interpret, draw and make

At level 4, a student:

4.7a Shows care in the shape, size and placement of parts when they match, make and copy things, including making nets of 3D shapes by drawing around their faces.

Evident when students, for example:

- Make a box recognisably— if not precisely— the same size and shape as a provided wooden block by drawing around each of its faces to make a net.
- Explain why their constructions did or did not work and how they might improve them (say they should have made tabs to hold the box together).
- Match faces of an object with parts of its net by visualising (colour the faces of a prism or pyramid and then, without folding, colour the parts of the net to match).
- Select and cut suitable lengths to make a same-size skeleton copy of a provided 3D shape (cut straws to the correct lengths to make a triangular prism).
- Use cubes to copy other structures made from cubes, attending to what cannot be seen but must be there.
- Make models of familiar and imaginary objects and scenes that, if not precise, are recognisable in shape, structure and scale (a television set, a netball ground, the surface of the moon).

4.7b Interprets and makes drawings of 3D shapes, using basic conventions for representing 3D space in 2D.

Evident when students, for example:

- Draw recognisable prisms, pyramids, cylinders and cones.
- Attempt to use some conventions of perspective or oblique drawing in their own work (drawing things farther away smaller, having parallel lines come closer together, using ellipses for circles, showing parallel edges with parallel lines).
- Use their understanding of some conventions when interpreting drawings (solid lines suggest edges that can be seen and dotted lines edges that cannot, that ellipses may be intended to suggest circles).
- Compare pictures produced in different cultures, commenting on how depth of perspective is shown.
- Construct models from visual information (origami or exploded drawings as for Lego).
- Select actual objects to match 2D representations in the form of exploded drawings or front, back and side views or various art forms (given a tray of objects, select one to match a drawing and explain why).

Visualising, analysing and representing arrangements and locations

At level 4, a student:

4.8 Visualises, follows and describes locations and paths and reads and makes maps and plans, using distance, direction, coordinates and scales.

Evident when students, for example:

- Give unambiguous instructions for moving and finding objects in their environment or on models, maps or plans, using distance, direction, angle multiples of 45°, compass points, coordinates.
- Instruct a computer to generate a specified simple shape using their knowledge of the shape's properties (rectangle).
- Produce maps and plans, labelling key features of a location or path according to the purpose of the map (provide a tour map of their school for visitors, a map of the location of some important workplaces in the community).
- Interpret maps relevant to their school, home and community to find their way around (use a plan of the library to find a particular book, a street directory to plan a journey).
- Find paths that meet specifications (the shortest or safest route, one which does not require retracing steps).
- Use simple ideas about scale in making and interpreting maps ('The map shows that the river is about ten times further away than the road', use a scale such as one centimetre for each metre).

Visualising, analysing and representing shapes

At level 4, a student:

4.9 Selects, describes and compares figures and objects on the basis of spatial features, using conventional geometric criteria.

Evident when students, for example:

- Choose geometric language with care to describe things clearly (use 'face' and 'edge' rather than 'side', say, 'Find me — I have 5 faces and 8 edges').
- Describe a diagram or shape so that a peer can reproduce or recognise it (describe a logo made of various figures to a friend over the telephone).
- Make figures and objects that meet criteria related to sides, faces, angles, edges (investigate geoboard triangles with two sides equal, with all sides equal).
- Visualise and select figures and objects that meet geometric criteria ('I am made from two triangles which are the same shape and size and three rectangles which are not the same').
- Link features of structures, such as flexible or rigid, fragile or strong, to their shape ('If I put a pop stick across from corner to corner it will make the shape rigid, because triangles are rigid').
- Explain why they think certain shapes will predominate in situations not yet investigated, such as a particular store cupboard, a location, a place (a storage locker, sideshow alley).

Visualising, analysing and representing movements and transformations

At level 4, a student:

4.10 Recognises and uses rotations, reflections and translations to relate the features of arrangements and patterns.

Evident when students, for example:

- Recognise and name rotations, reflections and translations and associated symmetries in patterns, objects and formations (in fabrics, logos, dances, tiles).
- Visualise and reproduce the folds and cuts used to make a complex symmetrical pattern (to copy a frieze, a snowflake).
- Describe the movement needed to show that one shape can be superimposed on another (turn it at right angles around the centre and slide it along).
- Decide whether a figure will tessellate and, provided with a template of it, make tiling patterns and describe the system of movements needed to continue the pattern.
- Consider whether certain types of shapes will tessellate and explain their thinking ('We think that all parallelograms will tile. This is because you can always slide them together to make long strips and then push the strips together.').
- Generate sequences of simple shapes that double or triple in dimensions by enlarging objects with blocks and figures with squares.

Level 5 outcomes:

5.7a Makes accurate versions of simple mathematical figures and objects, including making own nets.

5.7b Interprets and makes drawings of 3D shapes accurately, using conventions for representing 3D space in 2D with consistency.

5.8 Uses network diagrams to represent the order of, and paths between, locations and events.

5.9 Analyses, describes and uses distinguishing features of common classes of mathematical figures and objects, recognising parallels, perpendiculars and congruence.

5.10 Visualises, moves and sketches shapes to show the effect of translations, reflections, rotations and enlargements (using grids).

LEVEL 4 Number

Level 3 outcomes:

3.11 Counts, orders, estimates and describes with whole numbers, common fractions and decimal fractions for money or measurements (two places).

3.12 Identifies, continues and invents whole number patterns involving the four operations, including where successive terms in a sequence can be linked by an addition or subtraction strategy.

3.13 Uses understanding of whole and fractional numbers, relationships and operations to construct and complete simple statements of equality.

3.14 Makes a suitable choice of operation for situations involving whole numbers, amounts of money and familiar measurements.

3.15 Estimates and calculates mentally, including adding (sum to 100) and subtracting two-digit numbers and multiplying numbers to 10.

3.16 Uses understood written methods to add and subtract any whole numbers and amounts of money and to multiply and divide whole numbers by whole numbers to 10.

3.17 Uses a calculator for operating on whole numbers, amounts of money and measurements.

Count and order

At level 4, a student:

4.11 Counts, orders, estimates and describes with common and decimal fractions

Evident when students, for example:

- Read, write and say sentences expressed at least partly in the symbols for common and decimal fractions (⅛ of $300 = $60).
- Compare and order decimal fractions with both equal and unequal numbers of places (order given weight data for babies to two decimal places, order library books by their Dewey number).
- Use place value to explain why one decimal fraction is bigger or smaller than another (explain who is right in the following: 'When we put the books on the shelf, we put 65.6 before 65.125 because 6 is less than 125 but Tao and Karen said it wasn't right').
- Read scales calibrated in multiples of ten, where each calibration may not be labelled (read 3.97 m on a tape measure marked in hundredths but labelled in tenths).
- Count in decimal fractional amounts such as 0.3, 0.6, 0.9, 1.2... .
- Use the symbols =, < and > to order numbers and to state comparisons (write 6.75 < 6.9 and use 5 x $6 > 5 x $5.95 as a shorthand to explain why $30 is enough for five items at $5.95).
- Use readily visualised equivalences to compare and order fractions, understanding that fractions are relative to particular wholes (say that one quarter is less than three eighths because one quarter is the same as two eighths; explain one quarter of the extra large pizza is more than half of the medium pizza).

Number patterns

At level 4, a student:

4.12 Identifies, continues and invents whole and fractional number patterns, including those where successive terms in a sequence can be linked by a multiplication or division strategy.

Evident when students, for example:

- Find addition and subtraction patterns in rectangles in the one hundred chart (draw a box around four squares, add diagonally opposite numbers, do it for another box, what happens...?).
- Represent sequences of decimal fractions with materials or diagrams (show 0.3, 0.6, 0.9... by jumping along a number line; represent 3, 0.3, 0.03, 0.003 with MAB).
- Continue or fill in number sequences involving constant multiplication or division (2, 6, 18, 54,..., ...; 3,..., 27, 81,...).
- Follow a rule based on multiplication, division or simple fractions to generate a sequence (start with 64, then halve each time. Is 1 one of the numbers? What happens if you keep going?).
- Understand there will be a link between the way a sequence is built and the pattern of numbers generated (the first triangle has 3 matches and each time we need 2 more to get another triangle).
- Explain their conjectures about terms in a sequence by referring to the previous elements ('The pattern you are thinking of could be add one more each time because I worked out that the first jump was 3, then 4 then 5 then 6').
- Understand that the one string of three or four numbers can form part of any number of patterns (given a sequence beginning 1, 2, 4, explain that if the rule was 'multiply by 2', then the next two numbers would be 8 and 16; but if the pattern was 'add one more than the time before', then the next two numbers would be 7 and 11).

Equations

At level 4, a student:

4.13 Uses understanding of numbers and number relationships to construct and complete statements of equality, including those where more than one solution exists.

Evident when students, for example:

- Construct and verify complex arithmetic statements of equality ((4 x 3) + 10 – 2 = (7 x 2) + 6).
- Generate missing numbers that obey a constraint (to find ways of arranging 24 lamingtons into rectangular blocks, we need to find whole numbers so that _ x _ = 24, fill in (_ x _) x _ = 30 in as many ways as you can. Do you have them all?).
- Ask questions of clarification about the constraint ('When you ask us to find pairs of numbers that add to 10, can we use fractions?').
- Solve simple equations expressed in words ('I thought of a number, doubled it and added 1 and the answer was 17, what was the number?').
- Use number relationships to complete and justify statements without resorting to computation (use =, < or > in these sentences, 263 x 5 __ 120 x 10, 27 x 80 __ 54 x 40).
- Understand the relationship between division and fractions and use fractions to represent sharing situations where the divisor is larger than the dividend (sharing three pizzas between four people can be represented as 3 ÷ 4 or ¾).
- Investigate and record different ways of sharing an object or collection to generate equivalent statements (physically sharing 3 pizzas of the same type between four people could be done in several fair ways leading to ¾ = ¼ + ¼ + ¼ = ¼ + ½.

Applying numbers

At level 4, a student:

4.14 Makes a suitable choice of operation involving whole and fractional numbers (whole number multipliers and divisors), including those where more than one operation is needed.

Evident when students, for example:

- Select appropriate operations to deal with situations involving whole large numbers where intuition about the size of the solution may not help (to deal with 'Have you lived a million days?', 'How long ago was a million days?').
- Select appropriate operations to deal with situations involving decimal fractions, that is, whole number multipliers and divisors (calculate the cost of ruffle for a bedspread given dimensions and cost per metre of the ruffle).
- Select appropriate sequences of operations to solve problems where all the information needed is provided (provided with a page of advertisements, calculate costs for a shopping list).
- Restate problems symbolically in terms of the operation needed, including those involving common or decimal fractions (given 'I have three lengths of ribbon, each 4.5 m long to be cut into 9 pieces of equal length', write 4.5 ÷ 3 or (3 x 4.5) ÷ 9).
- Write 'stories' to represent symbolically expressed number sentences involving decimal and common fractions (describe a problem situation leading to the need to find 3.5 x 15).
- Plan and carry out a sensible sequence of calculations to determine quantities (use a memory capacity to make an efficient calculation of total costs or materials).
- Decide what needs to be calculated and therefore what extra information is needed to carry out a practical task (black edging costs $5.40 per metre, work out the cost of replacing the edge on your desk).

LEVEL 4 Number

Mental computation

At level 4, a student:

4.15 Estimates and calculates mentally, including adding and subtracting most two-digit numbers and multiplying and dividing multiples of 10 by one-digit numbers.

Evident when students, for example:

- Remember basic multiplication facts (to 10 x 10).
- Mentally add and subtract two-digit numbers.
- Mentally multiply one-digit numbers by multiples of ten ('9 x 70 is 9 lots of 7 tens so it's 63 tens, 630').
- Estimate sums and products by rounding to single digit multiples of 10 (456 + 572 is about 500 + 600 or 1100).
- Give upper and lower bounds to sums and products ('Each year is between 300 and 400 days and the dog has just turned 5 so she has lived between 1500 and 2000 days').
- Decompose double digit numbers to multiply and divide by small single digit numbers (3 twenty sixes is 3 twenties and 3 sixes, which is 60 + 18 = 78; 96 ÷ 3 is 90 ÷ 3 add 6 ÷ 3, which is 32).
- Mentally add fractions with like denominators commonly encountered, such as sum of proper fraction components (2¼ + 3¼).
- Use basic division facts to find a unit fraction of a whole number multiple (find one quarter of 28; one quarter of 25 is just over 6).

Written computation

At level 4, a student:

4.16 Uses understood written methods to add, subtract, multiply and divide whole numbers, money and measures (two places, whole number multipliers and divisors to 10).

Evident when students, for example:

- Add columns of decimal fractions with equal numbers of places (add 2.34, 11.21, 4.39).
- Subtract numbers with equal numbers of places (14.57 – 3.61).
- Multiply a decimal fraction by a one digit number (to find the perimeter of a regular pentagon, having measured and found that each side was 2.35 m).
- Divide amounts of money or measurements by a one digit number (to predict how much of their earnings each will get if four friends make $42.85 on their market stall).
- Use materials or some other means to provide a reasonable explanation of why their calculation method works (give an account such as 'For 127 ÷ 7, I thought of sharing the 12 tens first, that's one ten each and 5 tens left, so I had 57 left').
- Compare paper-and-pencil methods for ease, reliability and efficiency (compare the above method for finding 127 ÷ 7 with repeatedly subtracting 7 and with a conventional vertical algorithm).
- Use approximate additions, subtractions and multiplications to check that the decimal places in a calculation are reasonable.

Calculators

At level 4, a student:

4.17 Uses a calculator efficiently for operating on decimal numbers, including where more than one operation is needed, and interprets displays for division.

Evident when students, for example:

- Enter divisions and interpret remainders in the division process (enter 37 ÷ 3 to answer: Three children in a family shared $37, how much did each receive? Say what 12.333333 means, how much each child would get and how much over).
- Interpret negatives that appear when some subtractions are carried out (provide an informal explanation of the negative sign produced when they repeatedly subtract 7 from 27 to generate the pattern 27, 20, 13, 6, –1, –8; realise the significance of the negative sign if they enter 125 – 180 = in error to find how much more they need to raise if they have $125 and need $180).
- Plan sequences of calculations when a memory facility is used.
- Deal with brackets as required on their own calculator (enter (2.75 x 35) + (0.54 x 27) and (2.75 + 3.5) x (4.7 + 5.17) correctly on their own calculator).

Level 5 outcomes:

5.11 Interprets and uses whole powers and square roots and straightforward ratios and percentages.

5.12 (See outcome 5.28)

5.13 (See outcome 5.30)

5.14 Chooses and sequences several operations, covering situations involving decimal multipliers and divisors and division of smaller by larger numbers.

5.15 Estimates and calculates mentally with whole numbers, money and simple fractions, including multiplying and dividing some two-digit numbers by one-digit numbers.

5.16 Uses understood written methods to add, subtract, multiply and divide whole numbers and common and decimal fractions (whole number multipliers and divisors).

5.17 Makes efficient use of a calculator for common and decimal fractions and percentages and takes into account orders of operations.

LEVEL 4 Measurement

Level 3 outcomes:

3.18 Selects suitable and uniform things to use as units when measuring and a common unit to compare two things.

3.19 Directly and indirectly compares things, including by counting uniform units of angle, area, capacity and mass and measuring length to the nearest marked graduation.

3.20 Makes sensible numerical estimates using units that can be seen or handled and uses words such as 'between' to describe estimates.

3.21 Measures time and duration of time, reading clocks, calendars and straightforward timetables.

3.22 No outcome at this level

Choosing units

At level 4, a student:

4.18 Selects appropriate attributes and units of a sensible size for the descriptions and comparisons to be made.

Evident when students, for example:

- Understand that the same objects can be compared by different attributes and select attributes sensible for the purpose (compare two cartons by capacity if they are to store such things as marbles, but by lengths for packing books).
- Understand that several attributes must often be measured to describe and compare things adequately (to compare two people, height and mass might be described).
- Understand the effect of the size of the unit on the number needed to match an object (explain 'If I use small squares, I'll get a larger number than if I use big ones').
- Understand that a smaller unit will give a more accurate measurement (select as more accurate a smaller square over a larger square, a smaller triangle over a larger square, an eggcup over a litre).
- Show common sense in the choice of units (use hand spans for a first rough measure of whether a bookcase will fit a wall space).
- Select a unit of a suitable size to enable the comparison to be made (choose a smaller unit than a hand span upon realising that the bookcase and wall space are almost equal in size).

Measuring

At level 4, a student:

4.19 Measures and makes things, using conventional units and measuring equipment for length, mass, capacity and angle and reading scales to the nearest marked graduation.

Evident when students, for example:

- Read scales to the nearest graduation, including where the graduations may not be labelled (use a measuring jug with 50 mL graduations, and clock face (circular) scales with 100 grams marked).
- Compare and order length, capacity and mass measurements in common standard units (a 1400 metre walk is further than a one kilometre walk).
- Express measures of length, capacity and mass using common metric prefixes (kilo, centi, milli) and notation (mL, kg).
- Make things that meet simple measurement specifications in standard units ('Where could we fit a garden bed at least 6 metres long and 2 metres wide?').
- Find or make different things with one measurement the same but another different (each with a mass of 250 g but different volumes, each with an area of one square metre but different perimeters, each holding about a litre but different heights).
- Count cubes to measure the volume of a prism (show all the prisms that can be made with a particular number of cubes; copy and build prisms with cubes and order them by the number of cubes used).

Estimating

At level 4, a student:

4.20 Uses the known size of familiar things to help make and improve estimates, including those with centimetres, metres, kilograms and litres.

Evident when students, for example:

- Know the size of some familiar things they can use to help estimation (select which milk container holds a litre, say that a litre carton of milk weighs about a kilogram).
- Use the known size of common things to estimate size (heft a 500 g container of margarine and select things of similar mass).
- Use the known length of body parts and movements to help estimate length (know that the length of their average stride is 90 cm and use this to estimate the length of the oval).
- Estimate by looking lengths and areas to about 4 or 5 metres, centimetres, and square metres and simple fractions of these.
- Estimate by looking which containers have a capacity of about or less than or more than a litre and estimate by hefting which objects have a mass of about or less than or more than a kilogram.
- Consciously use feedback from their tests to improve progressive estimates in centimetres, metres, kilograms and litres (estimate the capacity in litres of a bucket, test, then estimate the capacity of another container by comparison with the first...).
- Collaborate to make sensible estimates of quantities (the quantity of soft drink needed for a party, the number of sheets of paper needed to cover the back wall, the time it will take for a journey, indicating what assumptions were made —such as about how much each person will drink).

Time

At level 4, a student:

4.21 Estimates and measures time and duration of time and prepares feasible timetables.

Evident when students, for example:

- Compare and use different calendars, such as Christian, Muslim, Aboriginal, Chinese, noting significant events on them (Muslim calendars are dated from when the prophet Mohammed fled Mecca, Christian calendars from the date of birth of Jesus Christ).
- Distinguish starting and finishing times from elapsed time (say the person who got up latest did not necessarily sleep longest).
- Use straightforward timetables and programs with both 12 and 24 hour times (read a TV program and convert to 24-hour time to set a video recorder).
- Prepare feasible timetables, criticising and improving their own and those of their peers.
- Identify factors that can distort estimates of time and estimate short spans of time such as half or one minute (clap for one minute, hold breath for 20 seconds).
- Use benchmarks to estimate quantities of time in minutes and hours (say, 'I don't think 5 minutes is reasonable. I take about 10 minutes to eat breakfast and I am sure the program ran longer.').
- Find out about and report on how different workers have to synchronise events (a cook has to get the food cooked at the same time).

Using relationships

At level 4, a student:

4.22 Understands and uses relationships involving perimeters of polygons and areas of regions based on squares.

Evident when students, for example:

- Recognise practical situations in which they need to find the perimeter of a region and calculate the perimeter by adding lengths.
- Devise and use their own short cuts for finding the perimeter of polygons (measure the length of two adjacent sides of rectangular garden, add and double, for a regular pentagon, multiply the length of one side by five).
- Decide whether a shape is close enough to rectangular that adding adjacent sides and doubling will be a 'good enough' estimate of perimeter for a task (deciding the amount of rope needed for an enclosure for the fete).
- Understand that shapes with the same area may have different perimeters and those with the same perimeter may have different areas (find which rectangle has the least perimeter for a fixed area).
- Use a systematic strategy to generate all the rectangles with a particular area in square units (make and justify that they have all the rectangles with an area of 36 squares, by referring to pairs of numbers whose product is 36).
- Investigate the areas of rectangles composed of squares and generalise that a short cut for finding the areas is to multiply the number in each row by the number of rows.

Level 5 outcomes:

5.18 Takes purpose and practicality into account when selecting attributes, units and instruments for measuring things.

5.19 Measures and makes things, using a range of graduated scales and strategies for making measurements that are more accurate than the available equipment allows.

5.20 Makes sensible estimates of length, area, mass and capacity in common standard units and identifies unreasonable estimates of things.

5.21 Estimates, measures and calculates time and duration of time and uses timelines and a range of types of timetable.

5.22 Understands and uses relationships involving length, area and volume, including those for shapes based on rectangles, triangles and rectangular prisms.

LEVEL 4 Chance and Data

Level 3 outcomes:

3.23 Distinguishes certain from uncertain things, describes familiar easily understood events as having equal chance of happening or being more or less likely.

3.24 Contributes to discussions to clarify what data would help answer particular questions or test predictions, and takes care in collecting data.

3.25 Classifies, sequences and tabulates data to help answer particular questions and varies the classification to answer different questions.

3.26 Displays and summarises data using frequencies, measurements and many-to-one correspondence between data and representation.

3.27 Reads and describes information in simple tables, diagrams, pictographs and bar graphs.

Understanding, estimating and measuring chance variation

At level 4, a student:

4.23 Places events in order from those least to those most likely to happen on the basis of numerical and other information about the events.

Evident when students, for example:

- Use available data to order things from least likely to most likely (using rainfall data to order capital cities from least to most likely to have rain in January).
- Order outcomes for a single random action from least to most likely by analysing the situation (for a die with faces 1, 1, 2, 2, 2, 3, state that a 2 is most likely, 1 is next and 3 is least likely).
- Order a probability device from the one most to least likely to produce an outcome (order three spinners with different proportions shaded yellow from the one least to most likely to produce a yellow result).
- Design a probability device, such as a die, spinner or bag of coloured beads, to produce a specified order of probability (colour a spinner with 8 segments so it would be most likely to stop on red, least to stop on green, and have the same chance of stopping on yellow as blue).
- Place informal expressions of chance on a scale from 0 to 1 (impossible, poor chance, even chance, good chance, and certain).

Collecting data

At level 4, a student:

4.24 Collaborates in deciding how data collection could help investigate situations and problems, frames helpful questions and decides what data to collect.

Evident when students, for example:

- Collaborate in identifying problems or areas of concern to their class and school (fair use of the school playgrounds and equipment, choice of a new school logo).
- Contribute to listing sub-questions to help investigate a general concern ('What equipment do we have and how much of each? What are the most popular activities? Who uses different parts of the school ground?').
- Identify data that could assist in dealing with a problem (find out favourite colours before designing a school logo).
- Suggest what data to collect to help estimate numbers or quantities (to estimate the number of children in a class when the class is at sport, count a number of different things in the classroom such as desks, raincoats, notebooks on teacher's table, to estimate how many raisins one could expect on average, collect data from a number of small packs of raisins).
- Revise a survey question so it can be answered by Yes/No or a simple multiple choice (begin with 'Which of these colours do you like?' and after trialling revise to 'Which pair of colours would you like best for our logo?').
- Design a test of their predictions about a probability device they have designed (a spinner that will come up red most often).

Organising data

At level 4, a student:

4.25 Classifies, sequences and tabulates data, with some grouping of data, and chooses methods helpful for answering particular questions.

Evident when students, for example:

- Construct and use their own categories to answer specific questions (for a project on animals in their area, they might ask how different animals move and decide to classify animals by 'walk', 'fly', 'wriggle' and 'swim').
- Suggest ways to improve a classification to better answer a question (realise that children riding a bicycle may have to cross the intersection alone and reclassify data into 'alone' or 'with older person').
- Suggest new questions arising from their organisation (classifying food by country of origin may lead to questions about which country's food is most popular amongst class members).
- Realise that it is sometimes helpful to group data, and group data involving whole numbers into class intervals (having estimated the number of sweets in a jar, with assistance organise the estimates in intervals such as 40-45, 46-50, 51-55, 56-60 to compare estimates with the true amount).
- Organise data in diagrams and tables that may include arrow diagrams, Venn diagrams, Carroll diagrams and two-way tables.

Displaying and summarising data

At level 4, a student:

4.26 Displays frequency and measurement data using simple scales on axes and summarises data with simple fractions, highest, lowest and middle scores, and means.

Evident when students, for example:

- Make quickly produced 'working' graphs in order to explore data (use stick-on notes with food choices written on them to make quick block graphs based on different ways of classifying the foods).
- Display data in bar graphs where the frequency axis may be scaled with multiples such as 0, 5, 10, 15... and where grouped measurement data may be treated as categories (graph frequencies for the groups 23-25, 26-28...).
- Use fractions to summarise data (about 0.4 of Year 2 ride to school, 0.3 walk, the rest come by car; the thumbtack fell on its side about two thirds of the time).
- Find the mean where there is enough data to make summarising sensible (the average height of the girls in class is 149 cm).
- Put data in order and describe the highest, lowest and middle scores (the heights of boys and girls in their class).
- Use a mean to get an estimate of a number (use the mean number of raisins in 25 boxes to estimate the mean generally).

Interpreting data

At level 4, a student:

4.27 Reads and describes information in tables (including some grouping of data), diagrams, line and bar graphs, fractions and means.

Evident when students, for example:

- Describe information from diagrams that may include arrow diagrams, tree diagrams, Venn diagrams or Carroll diagrams (this shows that 6 people like pizza but not hamburgers, 8 like hamburgers but not pizza, 2 don't like either, and 14 like both).
- Read the information provided on axes of bar and line graphs, including graduated scales.
- Interpret and report on information in tables and bar graphs where data is grouped into simple intervals that can be regarded as categories.
- Interpret and report on information in line graphs, informally describing trends in the data.
- Use the results of their data collection to comment upon their predictions ('We thought red would come up most, but red and green came up the same amount. We still think if we did it a lot, red would come up most.').

Level 5 outcomes:

5.23 Interprets and makes numerical statements of probability based on lists of equally likely outcomes and using fractions and percentages.

5.24 Collaborates in planning and refining survey questions and observation methods for collecting frequency and measurement data.

5.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, using class intervals and fields provided or planned with help.

5.26 Displays one-variable and two-variable data in plots and summarises data with fractions, percentages, means and medians.

5.27 Reads and describes information in histograms, plots and summary statistics and reports on data collection processes and results.

LEVEL FIVE

LEVEL 5 Statement

Students at level five can interpret and make accurate drawings of three-dimensional shapes. They show more precision in making models and can construct nets without tracing. They describe the geometric features of a collection of shapes and make more abstract generalisations about them. They follow instructions for moving or sketching things according to one or more transformations, and can make figures and objects with specified spatial characteristics. They read and produce network maps in common use in their communities, as well as produce their own network diagrams.

At this level, students are beginning to move easily between various ways of representing numbers and quantities. They order decimals with unequal numbers of places and read scales involving decimals. They can also use ratios in straightforward ways. They have a greater understanding of multiplication and division and know that these can increase or decrease a given quantity. They use a range of mental and written strategies for operations on whole numbers and common and decimal fractions. With multipliers and divisors of more than one digit, they use calculators. They use the strategy of estimation when it is sensible to do so rather than make unnecessary calculations.

Students are practical in their choice of units and equipment for measuring things, but may still need some prompting. They use a variety of graduated scales and experiment to find ways to get a measurement more accurate than the available instruments allow. They estimate, measure and calculate time and durations of time, and can interpret complex timetables and schedules if these are relevant to their lives. They can relate dimensions to areas and volumes of common two- and three-dimensional shapes.

Students at this level understand that probability statements give a measure of how likely something is to happen and can express probability using fractions and decimals. They plan surveys, observe experiments and generate data, organising it into diagrams, tables and spreadsheets. They use a variety of graphs to represent data, and confidently use means and medians when describing that data, and can extract information from a variety of sources.

Students produce rules describing how two quantities are related and express them in simple language, beginning to use shortened forms of the rule. They can find a number or numbers that satisfy constraints expressed in natural language and can interpret and draw informal sketch graphs relating two things.

Students at level five show a greater capacity to solve mathematical problems and use mathematical language.. They are systematic in thinking about key features of a problem, organising the information to make it easier to deal with and checking that their answers fit the specifications. They extend problems by posing their own questions and can explain some of the ways mathematics can and has been used to help us understand the world.

WORKING MATHEMATICALLY

SPACE

NUMBER

The description for this strand extends over 4 pages

MEASUREMENT

CHANCE AND DATA

ALGEBRA

LEVEL 5
Table of outcomes

5.1 Begins and extends tasks by asking some mathematical questions, including 'What would happen if...?'.
See page 82

5.2 Understands a conjecture as a guess with reasons and draws on mathematical knowledge to give reasons for conjectures before testing them.
See page 82

5.3 Uses problem-solving strategies, including those based on selecting and organising key information and being systematic.
See page 82

5.4 Checks that answers fit specifications and make sense in the original situation.
See page 83

5.5 Uses mathematical terms and notations with some care to describe objects and relationships and report conclusions with clarity.
See page 83

5.6 Explains some ways mathematics is used, or has been used in the past, to represent, describe and explain our world.
See page 83

5.7a Makes accurate versions of simple mathematical figures and objects, including making own nets.

5.7b Interprets and makes drawings of 3D shapes accurately, using conventions for representing 3D space in 2D with consistency.
See page 84

5.8 Uses network diagrams to represent the order of, and paths between, locations and events.
See page 84

5.9 Analyses, describes and uses distinguishing features of common classes of mathematical figures and objects, recognising parallels, perpendiculars and congruence.
See page 85

5.10 Visualises, moves and sketches shapes to show the effect of translations, reflections, rotations and enlargements (using grids).
See page 85

5.11 Interprets and uses whole powers and square roots and straightforward ratios and percentages.
See page 86

5.12 See outcome 5.28
See page 94

5.13 See outcome 5.30
See page 95

5.14 Chooses and sequences several operations, covering situations involving decimal multipliers and divisors and division of smaller by larger numbers.
See page 87

5.15 Estimates and calculates mentally with whole numbers, money and simple fractions, including multiplying and dividing some two-digit numbers by one-digit numbers.
See page 88

5.16 Uses understood written methods to add, subtract, multiply and divide whole numbers and common and decimal fractions (whole number multipliers and divisors).
See page 88

5.17 Makes efficient use of a calculator for common and decimal fractions and percentages and takes into account orders of operations.
See page 89

5.18 Takes purpose and practicality into account when selecting attributes, units and instruments for measuring things.
See page 90

5.19 Measures and makes things, using a range of graduated scales and strategies for making measurements that are more accurate than the available equipment allows.
See page 90

5.20 Makes sensible estimates of length, area, mass and capacity in common standard units and identifies unreasonable estimates of things.
See page 91

5.21 Estimates, measures and calculates time and duration of time and uses timelines and a range of types of timetable.
See page 91

5.22 Understands and uses relationships involving length, area and volume, including those for shapes based on rectangles, triangles and rectangular prisms.
See page 91

5.23 Interprets and makes numerical statements of probability based on lists of equally likely outcomes and using fractions and percentages.
See page 92

5.24 Collaborates in planning and refining survey questions and observation methods for collecting frequency and measurement data.
See page 92

5.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, using class intervals and fields provided or planned with help.
See page 93

5.26 Displays one-variable and two-variable data in plots and summarises data with fractions, percentages, means and medians.
See page 93

5.27 Reads and describes information in histograms, plots and summary statistics and reports on data collection processes and results.
See page 93

5.28 Follows and constructs rules for describing sequences and relationships between variable quantities using mostly natural language.
See page 94

5.29a Generates and plots data in first quadrant coordinate graphs, describing patterns in the resulting scatter of points.

5.29b Informally sketches and interprets graphs that describe the relationship between two quantities in everyday situations.
See page 94

5.30 Generates numbers or number pairs that satisfy a single constraint stated in natural language.
See page 95

LEVEL 5 Working Mathematically

Level 4 outcomes:

No outcomes at this level

Level 3 outcomes:

3.1 Poses mathematical questions prompted by similar or related questions or by specific stimulus materials.

3.2 Makes and tests conjectures including by responding to questions of the kind 'What would happen if...'.

3.3 Uses problem-solving strategies that include those based on selecting key information and representing it in models, diagrams and lists.

3.4 Uses a variety of ways when prompted to check working and choice of method.

3.5 Integrates terms and notations from space, number, measurement and chance and data into comparisons and descriptions of things.

3.6 Describes some of the mathematics of own and other cultures, past and present.

Investigating

At level 5, a student:

5.1 Begins and extends tasks by asking some mathematical questions, including 'What would happen if ...?' .

Evident when students, for example:

- Ask questions which begin 'What happens if ...' (having investigated their teacher's question 'What happens if you add two odd numbers?', they offer the question 'What happens if you add three odd numbers?').
- Change one condition in a mathematical problem to generate new mathematical questions (extend an activity on pentominoes by asking 'What if it wasn't five but six squares?' or 'What if it wasn't squares but triangles?' or 'What if it wasn't squares but cubes?').
- Suggest questions left unanswered by their mathematical work ('This method worked for all the triangles I tried, but I'm not sure what would happen if the triangle was obtuse', 'We found out what the boys preferred, but we can't be sure what girls prefer').
- Notice inconsistencies and raise questions when two approaches to the same task produce a different result ('I counted in lots of 0.2 and got 0.2, 0.4, 0.6, 0.8, 0.10, 0.12 ... but the calculator gave 0.2, 0.4, 0.6, 0.8, 1.0, 1.2... They can't both be right.').
- List and organise questions related to a practical task (to produce a weather vane, ask questions about shape [symmetries and balance], size, strength, support and placement, aesthetics; to choose between alternatives, list criteria such as cheapest, quickest, most likely).

Conjecturing

At level 5, a student:

5.2 Understands a conjecture as a guess with reasons and draws on mathematical knowledge to give reasons for conjectures before testing them.

Evident when students, for example:

- Give reasons for conjectures ('All parallelograms tile because you can slide them together to make strips and push the strips together. I think the regular pentagon will tile because all the regular shapes so far have.' [incorrect]).
- Comment upon their conjectures in light of results of testing their predictions and revise if needed ('I was wrong. The pentagon won't tile so some regular shapes tile and some don't.').
- Generate and tabulate examples and look for patterns in the data (find a number under 100 that has an odd number of factors, find another, another... Is there a pattern? Do I have them all?).
- Draw on their knowledge of numeration to explain addition and subtraction patterns in the one hundred chart (draw a box around four squares, add the diagonally opposite numbers, do it for another box, another... Identify and explain a pattern).
- Experiment with cuts and rearrangements of shapes to make and justify generalisations about related figures (express in their own words that the area of a triangle is half the area of a rectangle that has the same base and height and demonstrate why).
- Use graphs to search for relationships between variables and make predictions (to investigate 'Can you predict the area of a cube from its volume?', make cubes of side 1 cm, 2 cm, 3 cm... from squared paper, record surface area and volume, graph the ordered pairs, observe patterns and make conjectures).

Using problem-solving strategies

At level 5, a student:

5.3 Uses problem-solving strategies, including those based on selecting and organising key information and being systematic.

Evident when students, for example:

- Label information to make it easier to handle (make information from an advertisement giving car make, models and prices easier to handle by labelling the cars with letters).
- Use a strategy for moving around the information in a question (on labels write each person's name and mass and arrange labels to help solve a problem involving getting people across a river in boats with maximum loads).
- Simplify and organise information on problems or aspects of problems (use a topic web to cluster all aspects of a big problem, such as designing a proposal to reorganise the use of the playground).
- Systematically generate and list possibilities and explain why they think they have listed all possibilities.
- Use systematic approaches to eliminating possibilities and to improving when using guess and check (using a number line to the field of possible solutions to a 'What am I?' problem).
- Produce a systematic strategy for solving a problem based on progressively working towards a goal (to transfer people from one side of a river to another, develop a plan to get a number of people across after each set of moves).

Applying and verifying

At level 5, a student:

5.4 Checks that answers fit specifications and make sense in the original situation.

Evident when students, for example:

- Respond sensibly to questions such as 'What would a reasonable answer be?' and check that their answer is within the reasonable range (ask themselves, 'Is that a realistic size for a person?', 'Would a dinner cost that much?').
- Alter answers obtained mathematically to make them fit the realities of a situation by rounding appropriately, ignoring some possibilities (stating that the number of buses needed for an outing is 8 even though the calculator answer is 7.445...).
- Check results for accuracy at each stage of solving problems.
- Check their solutions against available information and draw general conclusions about their reasoning ('When I added the number of us who play hockey to the number who play tennis, I got more than there are in our class! I had forgotten that you can't just add when the two things being added overlap.').
- Check that their solution or resolution fulfils each of the original problem's givens and does the job (having a practice run on doughnut eating to test the way they plan to hang the doughnuts, checking that the carton they have made actually holds one litre).
- Comment sensibly on the suitability of criteria for a judgement of 'best' ('Is it sensible to buy two of these even if they are cheaper? Would we really want...?').

Using mathematical language

At level 5, a student:

5.5 Uses mathematical terms and notations with some care to describe objects and relationships and report conclusions with clarity.

Evident when students, for example:

- Relate symbolic mathematical expressions to their linguistic forms (% is per cent and > is greater than).
- Read and write sentences involving mathematical symbols ('We think this is better because 20% of $300 is $60').
- Choose geometric language with some care to be clear about meaning (use the words 'face' and 'edge' rather than 'side' when describing a 3D shape).
- Provide an oral or written description of a simple diagram, shape or position so that a listener or reader could reproduce or recognise it by the description alone.
- Clarify and revise verbal rules that describe the relationship between two quantities (compare 'double and add one' with 'add one and double').
- Provide an oral or written description of their work so that a listener or reader could understand clearly what mathematics was undertaken and the result.

Working in context

At level 5, a student:

5.6 Explains some ways mathematics is used, or has been used in the past, to represent, describe and explain our world.

Evident when students, for example:

- Describe a non-mathematical object or activity from a mathematical perspective (musical notes or an instrument, a football or netball game, the shape of fish or leaves, perspective or surrealism in art, basket weaving, a dance, darts).
- Observe and explain number sequences in the features of natural or built things (16, 11, 8, 5.6 ... in photography).
- Use mathematics within their repertoire to help explain a phenomenon or object in their environment (why bees' cells are hexagonal, how a kaleidoscope works).
- Describe the basic mathematics in some visual representations of physical aspects of our world (compare a globe with two different map projections and decide whether direction, length, parallelism and area are preserved; describe how betweenness, order, orientation, proximity and scale are or are not represented on a schematic map —or network— such as those associated with the Dreaming or suburban bus and train routes).
- Find out about and report accurately on the meaning of numbers as used in their community to label and describe (3-iron, 20-20 vision, ISBN 0-460-02402-7, ASA 400, 10 point type).
- Find out about some different models of the universe from other times and cultures and describe some of the simpler mathematics they involve.

Level 6 outcomes:

No outcomes at this level

Level 7 outcomes:

7.1 Poses, clarifies and refines mathematical questions to help understand or guide the investigation of a situation.

7.2 Makes generalisations by abstracting common mathematical features from situations, tests with additional cases and explains why generalisations must be true.

7.3 Uses problem-solving strategies that include identifying and working on related problems or sub-problems.

7.4 Applies standard methods or models, comparing and choosing between alternatives, including by considering assumptions needed and results obtained.

7.5 Uses conventional mathematical language to help give clear and logical accounts of mathematical work.

7.6 Makes links between the development and use of mathematical ideas and the conditions and concerns of the individuals and communities that produce them.

LEVEL 5 Space

Level 4 outcomes:

4.7a Shows care in the shape, size and placement of parts when they match, make and copy things, including making nets of 3D shapes by drawing around their faces.

4.7b Interprets and makes drawings of 3D shapes, using basic conventions for representing 3D space in 2D.

4.8 Visualises, follows and describes locations and paths and reads and makes maps and plans, using distance, direction, coordinates and scales.

4.9 Selects, describes and compares figures and objects on the basis of spatial features, using conventional geometric criteria.

4.10 Recognises and uses rotations, reflections and translations to relate the features of arrangements and patterns.

Using spatial ideas, tools and techniques to interpret, draw and make

At level 5, a student:

5.7a Makes accurate versions of simple mathematical figures and objects, including making own nets.

Evident when students, for example:

- Make nets composed of rectangles, triangles or circles to construct a named 3D model or a simple shape they have in mind, anticipating the level of precision needed for the model to be correct in form and size.
- Consider in advance how a model will be held together (placement of tabs on a net of a solid).
- Attend to essential details when constructing figures and objects (matching lengths and angles, ensuring the hole is in the centre of the circle, the paper is folded exactly on the diagonal).
- Visualise which of a series of possible nets —that is, figures made by combining faces of the right shapes and number— will fold to make a particular model.
- Predict which hexominoes can be used as nets for a cube and visualise the relationship of the squares to each other in the folded cube (colour the faces of a cube different colours, and, without folding, indicate where the colours could be on the net).
- Make complex 3D models by combining models of pyramids, prisms and other simple shapes.

5.7b Interprets and makes drawings of 3D shapes accurately, using conventions for representing 3D space in 2D with consistency.

Evident when students, for example:

- Describe and compare common representations of objects and spaces (computer images, weather and contour maps, clothing patterns and exploded drawings for kit assembly).
- Use spatial language to describe and compare oblique, perspective and isometric drawings (explain that for oblique drawings parallel edges are shown with parallel lines, while in perspective the lines come closer together).
- Match objects with 2D representations of them (produce a complex shape made with cubes on the basis of an isometric drawing or front, back and side view, considering what cannot be seen but must be there).
- Visualise an object or scene in different orientations and draw possible other views of an object from information contained in 2D drawings.
- Use with some consistency basic conventions for oblique or perspective drawing.
- Draw an arrangement of objects made from cubes using isometric paper and from front, side and back views (draw a Soma cube piece from direct observation).

Visualising, analysing and representing arrangements and locations

At level 5, a student:

5.8 Uses network diagrams to represent the order of, and paths between, locations and events.

Evident when students, for example:

- Identify the physical features of a situation to be represented in a network (rooms in the mathematics block as nodes and doors as connecting paths).
- Draw network diagrams to represent familiar locations or based on information taken from a conventional map (plan bus routes to connect a new housing estate to other areas).
- Draw networks based on observation of procedures (watch a carpenter make an object and represent the sequence of actions, represent locations visited regularly by a community health worker).
- Represent non-physical features of a situation as a network (individuals as nodes and family connection as paths).
- Interpret network diagrams —railways, airlines—used in their community (planning a day trip for the school during their visit to the capital).
- Identify best paths and locations according to various criteria (start and finish at the bus stop without retracing one's steps, locate a recycling depot, design a tour of country towns by a rock group), indicating strengths and weaknesses of alternatives.

Visualising, analysing and representing shapes

At level 5, a student:

5.9 Analyses, describes and uses distinguishing features of common classes of mathematical figures and objects, recognising parallels, perpendiculars and congruence.

Evident when students, for example:

- Describe features that distinguish one common class of shapes from another ('Prisms have two congruent parallel faces but pyramids don't').
- Understand and describe 'inclusive' relationships between common classes of shapes (show parallelograms, rectangles and squares in a Venn diagram and say 'All squares are rhombuses, but the reverse isn't true').
- Generate and classify shapes that satisfy a given condition (use straws to produce shapes with two equal diagonals. What do they have in common? What if we add the constraint that the diagonals cross at their mid-points?).
- Identify and name parallel and perpendicular lines and planes in figures and objects (in a triangular prism).
- Describe the common features of collections of shapes they can see and handle (the total number of vertices on each of our prisms is twice the number of vertices on one of the pair of parallel faces).
- Suggest difficulties with descriptions of shapes ('It's true that triangular pyramids [tetrahedrons] are made up completely of triangles but just saying the shape has triangular faces isn't enough. This shape is made up of triangles but it isn't a pyramid.').

Visualising, analysing and representing movements and transformations

At level 5, a student:

5.10 Visualises, moves and sketches shapes to show the effect of translations, reflections, rotations and enlargements (using grids).

Evident when students, for example:

- Visualise to decide whether two figures or objects are congruent, and if so, whether a reflection is needed.
- Make designs that exhibit specified (rotational, reflection, translation) symmetries (fold paper, mirrors, computer graphics).
- Use reflections, translations or rotations described in natural language to follow and give instructions for moving objects (to walk out a shape, produce a spiral using LOGO).
- Investigate and report on the effect of translations, rotations or reflections on the position and orientation of figures (use a Mira mirror to sketch the effect of reflecting a drawing through a line and report, 'We found that each bit of the reflection was as far behind the Mira as that bit of the figure was in front').
- Use transformations to modify tessellating shapes to produce other tessellating shapes and informally explain why they work (Escher-type designs based on rectangles).
- Plan their own use of grids to enlarge and reduce figures (by whole number and unit fraction scales) and to make distortions (double widths but not heights).
- Enlarge models made with cubes to a small whole number scale (given a model made of 6 cubes, like a Soma cube piece, produce one enlarged by a scale factor of 3).

Level 6 outcomes:

6.7 Uses geometric techniques and tools to interpret and meet specifications requiring the accurate construction and placement of figures and objects.

6.8 Visualises, sketches and describes paths and regions that satisfy conditions given in everyday language.

6.9 Analyses, describes and uses relationships in and between classes of figures, including parallel, perpendicular and intersecting lines, triangles and rectangles.

6.10 Visualises, produces and accurately describes translations, reflections, rotations and enlargements.

LEVEL 5 Number

Level 4 outcomes:

4.11 Counts, orders, estimates and describes with common and decimal fractions.

4.12 Identifies, continues and invents whole and fractional number patterns, including those where successive terms in a sequence can be linked by a multiplication or division strategy.

4.13 Uses understanding of numbers and number relationships to construct and complete statements of equality, including those where more than one solution exists.

4.14 Makes a suitable choice of operation involving whole and fractional numbers (whole number multipliers and divisors), including those where more than one operation is needed.

4.15 Estimates and calculates mentally, including adding and subtracting most two-digit numbers and multiplying and dividing multiples of 10 by one-digit numbers.

4.16 Uses understood written methods to add, subtract, multiply and divide whole numbers, money and measures (two places, whole number multipliers and divisors to 10).

4.17 Uses a calculator efficiently for operating on decimal numbers, including where more than one operation is needed, and interprets displays for division.

Count and order

At level 5, a student:

5.11 Interprets and uses whole powers and square roots and straightforward ratios and percentages.

Evident when students, for example:

- Use unitary ratios (of the form '1 part to b parts') when making comparisons of parts within a whole (say, 'In the first lot the ratio of cordial to water was 1 to 4. That was too strong, so I added another lot of water which made it 1 to 5.').
- Understand that common fractions are used to describe ratios of parts to whole (that is, ratios of the form 'a parts in b') and use to make comparisons (say, 'One-fifth of this is cordial but only one sixth of that is'; to estimate the size of a collection, say, 'About two in five people ride to school. If the school grows to 550 students there will probably be about 220 bikes.').
- Interpret and use percentages to make straightforward comparisons (say, 'This morning, I got 26 balls from 50 tries, that's 52%. This afternoon, I got 24 from 40 tries, that's 60%. I may be improving a bit.').
- Use common equivalences between decimal and common fractions and percentages when comparing quantities ('One-third off is better than 30% discount').
- Use whole number powers and square roots in describing things (use and evaluate 4^3 for the volume of a cube of size 4 cm, $\sqrt{225}$ for the length of the side of a square of area 225 sq cm).

Number patterns

5.12 See outcome 5.28 in Algebra.

Equations

5.13 **See outcome 5.30 in Algebra.**

Applying numbers

At level 5, a student:

5.14 Chooses and sequences several operations, covering situations involving decimal multipliers and divisors and division of smaller by larger numbers.

Evident when students, for example:

- Use multiplication in calculating amounts from simple familiar rates such as price and for areas and cross products (multiply as a short cut to estimate the area of a playing field roughly rectangular and about 105 [metre] strides by 220 [metre] strides).
- Recognise the need to multiply where the multiplier is a decimal fraction greater than one (find about how much time someone will take to run 2.75 km if they can run a kilometre in about 4.5 minutes).
- Recognise the need to multiply where the multiplier is a decimal fraction less than one (decide how tall a rectangle will be if the dimensions are reduced to three quarters).
- Recognise the need to divide where the divisor is a decimal fraction greater than one and less than one, respectively (to find about how many 1.75 litre containers can be filled from a 40 litre container or how many pieces 0.4 metre long can be cut from 10 metres of fabric).
- Use division when dividing a smaller number by a larger number (find how long each piece of tape would be if we cut a 5 m length into 8 equal pieces).
- Select the appropriate division to deal with familiar everyday rates such as price and speed (we can average about 80 kilometres per hour and the towns are 450 km apart. How long will it take us?).

LEVEL 5 Number

Mental computation

At level 5, a student:

5.15 Estimates and calculates mentally with whole numbers, money and simple fractions, including multiplying and dividing some two-digit numbers by one-digit numbers.

Evident when students, for example:

- Decompose double digit numbers to multiply them by single digit numbers (6 twenty-sixes is 6 twenties and 6 sixes, which is 120 and 36 or 156).
- Add fractions using well-known equivalences where the proper fraction parts do not add to more than 1, such as $2\frac{1}{4} + 1\frac{1}{2}$.
- Subtract fractions using well-known equivalences, such as $\frac{3}{4} - \frac{1}{2}$.
- Calculate unitary fractions of whole number and decimal amounts (find one-seventh of 45, estimate one-fifth of 8.76 metres).
- Calculate durations of time less than one hour ('It is 3.45 pm. How long until 4.20 pm?').
- Use front-end estimation ($16.67 + $4.12 + $0.97 + $0.46 is more than $16 + $4 + $0 + $0, that is $20; $3\frac{1}{3} + 2\frac{1}{2}$ is more than 5).
- Round to the nearest (state that 16.67 + 4.12 + 0.97 + 0.46 is about 17 + 4 + 1 + 0 = 22).

Written computation

At level 5, a student:

5.16 Uses understood written methods to add, subtract, multiply and divide whole numbers and common and decimal fractions (whole number multipliers and divisors).

Evident when students, for example:

- Add columns of decimal fractions, including those with unequal numbers of places (add 2.343, 11.21, 4.391).
- Subtract numbers, including those with unequal numbers of places (write 14.57 – 3.613 as 14.570 – 3.613).
- Use informal written methods to multiply by two digit numbers (visualise or sketch a rectangular model and separate tens and units; note that 27 lots of 34 is 20 lots of 34 plus 7 lots of 34, which is 2 x 340 + 7 x 34 =...).
- Divide decimals by one-digit numbers, interpreting remainders and deciding whether to round up or down.
- Record stages in finding proper fractions and percentages of quantities they cannot complete mentally (to find $\frac{3}{4}$ of $48, find one quarter mentally, record $12 and multiply by 3).
- Record stages in adding and subtracting fractions they cannot complete mentally (write $2\frac{3}{4} + 3\frac{1}{2} = 5 + \frac{3}{4} + \frac{1}{2} = 5 + 1\frac{1}{4} = 6\frac{1}{4}$, write $2\frac{3}{4} - 1\frac{1}{2} = 1\frac{3}{4} - \frac{1}{2} = 1\frac{1}{4}$).

Calculators

At level 5, a student:

5.17 Makes efficient use of a calculator for common and decimal fractions and percentages and takes into account orders of operations.

Evident when students, for example:

- Use a calculator to investigate the effect of multiplying and dividing by fractional amounts (play Target, the goal of which is to get to 100 by multiplying only, and explain that multiplying by a number less than one makes smaller).
- Compare orders of operations on different calculators (to evaluate (2.75 x 35) + (0.54 x 27) and (2.75 + 3.5) x (4.7 + 5.17)).
- Convert between fractions, decimals and percentages (enter five-eighths into a calculator as 5 ÷ 8 to find the decimal equivalent).
- Use a calculator to express one quantity as a percentage of another and to find fractions and percentages of numbers.
- Enter fractions in calculators that use a common fraction format.
- Report on different ways to increase or decrease a quantity by a given percentage (compare finding 15% and subtracting from an original amount with finding 85%).

Level 6 outcomes:

6.11 Interprets and uses whole powers and roots, scientific notation, ratios, percentages and negative numbers.

6.12 (See outcome 6.28)

6.13 (See outcome 6.30)

6.14 Uses a ratio or familiar rate to describe the relationship between two directly proportional quantities and to calculate one quantity from another.

6.15 Estimates and calculates mentally with whole and fractional numbers, including finding frequently used fractions and percentages of amounts.

6.16 Uses understood written methods to calculate with decimal and common fractions and integer powers.

6.17 Makes efficient use of a scientific calculator, including for powers and roots and using scientific notation.

LEVEL 5 Measurement

Level 4 outcomes:

4.18 Selects appropriate attributes and units of a sensible size for the descriptions and comparisons to be made.

4.19 Measures and makes things, using conventional units and measuring equipment for length, mass, capacity and angle and reading scales to the nearest marked graduation.

4.20 Uses the known size of familiar things to help make and improve estimates, including those with centimetres, metres, kilograms and litres.

4.21 Estimates and measures time and duration of time and prepares feasible timetables.

4.22 Understands and uses relationships involving perimeters of polygons and areas of regions based on squares.

Choosing units

At level 5, a student:

5.18 Takes purpose and practicality into account when selecting attributes, units and instruments for measuring things.

Evident when students, for example:

- Select a unit practical for the purpose of the measurement (measures to the nearest metre to produce a map with a 1:1000 scale).
- Select a level of accuracy suited to the importance of the measurement (a rough pacing out of the pitch will do for a friendly game of cricket but not for a weekend competition game).
- Understand that the choice of a unit depends upon the fineness of the comparisons to be made (the need for accuracy) rather than simply the size of the object to be measured (a bridge may be measured in metres to estimate paint needed but in millimetres for engineering work).
- Choose appropriate instruments and explain their choice (you need scales to measure the mass of ingredients but bathroom scales would not be accurate enough for recipes; a trundle wheel is more accurate than a metre rule for measuring around a curved flower bed).
- Understand that standard units are not in themselves more accurate or precise than non-standard units but enable us to communicate and record measurements with precision.

Measuring

At level 5, a student:

5.19 Measures and makes things, using a range of graduated scales and strategies for making measurements that are more accurate than the available equipment allows.

Evident when students, for example:

- Read scales where there are five or twenty sections marked between units and each calibration may not be labelled (calibrated to represent 1.0, 1.2 , 1.4, 1.6, 1.8, 2.0... but labelled only with 1 and 2).
- Use instruments correctly and accurately (place a measuring jug on a level surface and read with markings at eye level to reduce reading error).
- Find and use ways of accurately measuring objects too big or too small for the available equipment (estimating the volume of a drop of water from the tap, the thickness of a sheet of paper).
- Understand capacity as a measure of the internal volume of a container and relate units of volume to units of capacity (also called liquid volume).
- Use liquid displacement to measure and order objects by their volume and find or make a variety of things with a volume of about a cubic metre, cubic decimetre, cubic centimetre.
- Measure and make angles to a specified size using a protractor or other equipment, accurately to within 5°.

Estimating

At level 5, a student:

5.20 Makes sensible estimates of length, area, mass and capacity in common standard units and identifies unreasonable estimates of things.

Evident when students, for example:

- Judge the size of common quantities (find lengths of about 1 mm, 1 cm, 1 m; capacities of about 1 L, 250 mL, 25 mL; areas of about 1 sq. cm, 1 sq. m; masses of about 1 kg, 100 g).
- Make standard unit estimates of length, capacity and mass by comparing with known quantities (the ceiling is about 3 m high because the cupboard is about 1 m and 3 would reach the ceiling).
- Identify and use reference points for making and judging estimates (dimensions and area of an A4 sheet of paper, Australia's yearly consumption of petrol).
- Improve over progressive estimates (heft to estimate the mass in grams of a cup of flour, use a scale to check, compare a cup of rice with the flour by hefting, estimate the mass of the rice).
- Recognise unrealistic measures of familiar things (a classmate is unlikely to weigh 5 kg).
- Judge whether an estimate is reasonable by comparison with a known measure (say, 'The room can't be 8 metres long because my stride is less than a metre and the room is only 7 strides long').

Time

At level 5, a student:

5.21 Estimates, measures and calculates time and duration of time and uses timelines and a range of types of timetable.

Evident when students, for example:

- Compare different ways people measure time intervals, both natural (sun, human life cycle, tides) and artificial (seconds, 24 hour day) units of time and associated means of measurement.
- Produce a timeline of significant dates for Australian people, a sport, the continent or a species (known dates of Aboriginal habitation of Australia, life of Jesus, life of Mohammed, Dutch landing, Federation).
- Estimate time from starting and finishing times (decide if a 3-hour video will last from 11.35 pm. to 2.50 am.).
- Compare starting and finishing times on clocks, calendars and timetables to find how long something will take or has taken (calculate how long the lunch break is from reading the timetable).
- Calculate elapsed time to compare events (the express bus arrives a bit later but only takes 40 minutes, whereas the regular bus takes 55 minutes).
- Estimate time from simple rates involving times (time of arrival for journeys based on average speed, time a tap has dripped based on total water loss and rate of loss).
- Interpret local timetables and schedules (bus or train timetables, tide sheets, breeding tables).

Using relationships

At level 5, a student:

5.22 Understands and uses relationships involving length, area and volume, including those for shapes based on rectangles, triangles and rectangular prisms.

Evident when students, for example:

- Recognise practical situations where they need to find the area of regions and devise short cuts for doing so (to calculate advertising revenue for a local paper, given the charges for advertising, based on areas, and an example of the paper).
- Decide on a minimal set of measurements needed to calculate perimeters and areas of shapes based on rectangles, and surface areas and volumes of shapes based on rectangular prisms.
- Inspect the nets of various rectangular prisms and generate short cuts for finding their surface areas.
- Recognise practical situations in which they need to find the volume of an object and calculate the volume of rectangular prisms.
- Demonstrate suitable dissections of complex shapes into several rectangles or rectangular prisms for each of which the dimensions, and hence the area and volume, can be determined.
- Estimate areas of various quadrilaterals by counting squares and make conjectures relating lengths and areas (make different parallelograms on a geoboard, count squares, record data on base, height and area, make conjectures relating parallelograms to rectangles).

Level 6 outcomes:

6.18 & 6.19 Decides what measurements are needed to complete a practical task and how to obtain them and make or collect measurements to the planned level of accuracy.

6.20 Unprompted, estimates in situations in which it is sensible to do so and judges the reasonableness of estimates.

6.21 Integrates information from several sources to determine time and duration of time and to plan and synchronise events.

6.22a Understands and uses relationships involving length, area and volume of quadrilaterals and circles, prisms and pyramids.

6.22b Understands and uses similarity and Pythagoras's theorem to solve problems involving right triangles and scale drawing.

LEVEL 5 Chance and Data

Level 4 outcomes:

4.23 Places events in order from those least to most likely to happen on the basis of numerical and other information about the events.

4.24 Collaborates in deciding how data collection could help investigate situations and problems, frames helpful questions and decides what data to collect.

4.25 Classifies, sequences and tabulates data, with some grouping of data, and chooses methods helpful for answering particular questions.

4.26 Displays frequency and measurement data using simple scales on axes and summarises data with simple fractions, highest, lowest and middle scores, and means.

4.27 Reads and describes information in tables (including some grouping of data), diagrams, line and bar graphs, fractions and means.

Understanding, estimating and measuring chance variation

At level 5, a student:

5.23 Interprets and makes numerical statements of probability based on lists of equally likely outcomes and using fractions and percentages.

Evident when students, for example:

- Understand that we describe events that cannot happen as having a probability of 0, events that are certain as having probability of 1, and events that may happen as having a probability between 0 and 1, depending upon how likely they are to occur.
- Interpret expressions of probability in general usage such as 'The probability of rain tomorrow is 30%' and 'There's a fifty-fifty chance' (if the probability of a child getting a cold this winter is 15% and falling off a bike is 20%, understand that the latter is more likely).
- List equally likely outcomes for a one-step action to assign probabilities (each of 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are equally likely to appear as the last digit of a telephone number —but not as the first digit— so the probability that the last digit is 7 is 0.1).
- Use fractions to assign probabilities (14 girls' and 16 boys' names are put in a bag: the probability of drawing a boy's name is 16/30).
- Use the results of a simple experiment to predict the results of a repetition of it (use results of tossing a hockey stick 50 times to predict what would happen in a later experiment of 100 tosses).
- Design a device to fit specified probabilities (colour a spinner so the probability of it stopping on red is 0.5, on green is 0.1, and on blue is 0.4).

Collecting data

At level 5, a student:

5.24 Collaborates in planning and refining survey questions and observation methods for collecting frequency and measurement data.

Evident when students, for example:

- Plan data collection to enable the comparison of two groups on the same variables (when researching 'Do girls and boys in our year level like the same...?', plan data collection sheets with a place to record sex of respondent).
- Collaborate in developing and trialling a short series of questions (two or three questions) involving Yes/No answers, simple multiple choice responses, or categories, such as 'Do you think teenagers should have to help around the home for pocket money?' (Yes/No).
- Collaborate in making and refining data collection sheets involving lists or tables or scales (decide which categories to use in investigating animal behaviour for science project).
- Plan and improve ways to collect measurements consistently (how to mark off a pace so that each one is done the same way, how to use the stopwatch reliably).
- Plan ways to collect frequency information consistently (decide what counts as a Yes to a test for tongue curling, how to ensure everyone records food preferences in the same way).
- Carry out data collection consistently and accurately (ask the planned question in the same way each time, measure between the same two body parts, round in the same way, use categories consistently in a tally sheet).

Organising data

At level 5, a student:

5.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, using class intervals and fields provided or planned with help.

Evident when students, for example:

- Collaborate in planning how to organise measurement data to answer specific questions (to organise a class set of attempts to estimate a time interval to help answer the question 'How good are our estimates?').
- Contribute to discussions about how to collapse data to help 'get a feeling for it' (suggest placing estimates written on stick-on slips of a time interval into groups such as 23-25, 26-28, 29-31, 32-34, 35-38, and use to produce quick graphs to compare estimates with the true time).
- Use data to generate new questions (comparing their grouped estimates with the true measure could lead to questions about how graphs of estimates could help give better estimates).
- Organise measurements in tables with provided class intervals (find the mass of various rocks and group the data from 0 up to 250 g, more than 250 g and up to 500 g... (or $0\ g \leq m < 250\ g$...).
- Enter data in databases with fields already defined (enter information on the books each classmate reads in preparation for a Bookweek display).

Displaying and summarising data

At level 5, a student:

5.26 Displays one-variable and two-variable data in plots and summarises data with fractions, percentages, means and medians.

Evident when students, for example:

- Display one-variable data in line plots with various scales including multiples and decimal fractions (make a line plot of the actual time span for each person's estimate of 30 seconds).
- Use stem plots to group and display one-variable data (make a stem plot of the ages of their parents or caregivers).
- Represent two-variable data in scatter plots and make informal statements about relationships (about the relationship between time spent reading and watching TV each week).
- Use means or medians to summarise data where there is enough to make summarising sensible (of the number of hours class members spend watching television).
- Understand different ways in which the word 'average' is used and the advantages and disadvantages of the mean, median and mode.
- Use fractions or percentages to compare data ('Before, I got 26 balls from 50 tries, that's 52%. This time I got 24 from 40 tries, or 60%. They are close, but I may be improving a bit.').

Interpreting data

At level 5, a student:

5.27 Reads and describes information in histograms, plots and summary statistics and reports on data collection processes and results.

Evident when students, for example:

- Interpret bar graphs and histograms for grouped data, including where the scales on the axes must be read between calibrations.
- Informally interpret relationships and reach conclusions from scatter plots ('It looks like the people who read fastest usually read the most, but we can't tell which one causes the other').
- Write or present an accurate summary of the information displayed in a range of tables and graphs (what is shown by their line plot of ages of the caregivers of class members).
- Present short written or oral reports of their surveys, describing initial questions, data collection processes and conclusions, and commenting on how they might be improved.
- Check the accuracy of their data before interpreting it ('We plotted the areas of the cubes against the volume of the cubes, and found that we could draw a smooth curve through all the points except one, so we checked and found we'd made a mistake').

Level 6 outcomes:

6.23 Estimates probabilities and proportions based on primary or secondary data collection and assigns probabilities for one- and two-stage events by reasoning about equally likely events.

6.24 Plans experiments and surveys, collaboratively and independently, deciding how to collect data consistently and the sources and types of data.

6.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, planning class intervals and fields collaboratively and individually.

6.26 Displays and summarises data to show location and variability (including when some grouping of data is required) to compare data sets and to show relationships in one data set.

6.27 Interprets collected and published data from tables, diagrams, plots, graphs, prose and databases to make comparisons and describe relationships.

LEVEL 5 Algebra

Levels 1–4
Algebra is not included in these levels.

Expressing generality

At level 5, a student:

5.28 Follows and constructs rules for describing sequences and relationships between variable quantities using mostly natural language.

Evident when students, for example:

- Follow rules that describe, in natural language, how to get an element in a pattern from the previous element or elements (start with this rectangle and build a sequence of them, each having double the dimensions of the previous one).
- Identify a pattern in a sequence and describe a rule for generating an element of a sequence from the previous element (the rule is 'Start with $100 and add 50% each month').
- Follow rules that describe, in natural language, a relationship between two variables ('Square your number and then add 2', or 'The cost is $20 deposit plus $4 per hour').
- Find a general rule that relates each element of a sequence based on one or two operations to its position (say, 'The first number was three times 1 add one, the second number was three times 2 add one, the third number... Each number is three times its position add one, so if that's the rule the 20th number would be three times 20 add one, which is 61.').
- Rewrite rules expressed in natural language in shortened form to make them easier to follow, using brackets or the rule of order as needed (each number in the sequence is (3 x position number) + 1; or (3 x n) + 1 where n is the number of the position).
- Compare and revise rules to make clear their meaning (play 'Guess my rule' and compare 'double and then add 1' with 'add 1 and then double' as descriptions of the rule, writing each expression in shorthand: (number x 2) + 1 and (number + 1) x 2 or (n x 2) + 1 and (n +1) x 2).
- Write clear instructions to generate an invented sequence using words, flow charts or diagrams, or a computer language (a BASIC program to generate the odd whole numbers).

Function

At level 5, a student:

5.29a Generates and plots data in first quadrant coordinate graphs, describing patterns in the resulting scatter of points.

Evident when students, for example:

- Locate ordered pairs of values on a coordinate grid and describe the position of points on a grid using ordered pairs (describe the location of a marked point as (5, 4)).
- Interpret the points on a coordinate grid in terms of the variables graphed (state that after 60 seconds the water level was 5 cm, match Australian cities with points plotted on a coordinate grid with population on one axis and distance from Adelaide on the other).
- Generate or collect data in the form of pairs of numbers or quantities (measure to find pairs of values for the diameter and circumference of circular lids; heights of plants over time).
- Draw and describe scatter graphs by plotting ordered pairs on simple scaled axes (graph heights and arm lengths of class members and informally comment on whether there appears to be any relationship).
- Recognise situations where it does or does not make sense to join graphed points (it does not make sense to join the points on a graph of the price of different numbers of bubblegum).
- Describe the situation summarised in their graph ('The diameter of a lid appears to be related to its circumference. We could use the graph to predict the circumference from the diameter.').

5.29b Informally sketches and interprets graphs that describe the relationship between two quantities in everyday situations.

Evident when students, for example:

- Identify the variables in a situation based on a description or familiarity (give the variables in the amount of daylight in a typical day changes over the year; 'the amount of pancake mixture needed is related to how many people we want to feed').
- Use the relative positions of two points on a graph, rather than a detailed scale, to compare the things they represent (from a point graph of the ages and heights of Marta and Ali say, 'Marta is older than Ali and Marta is shorter than Ali').
- Tell the story shown by a graph by describing how one quantity varies with the other (a graph that shows how the noise level in a classroom changed over a period).
- Compare alternative graphs of the same easily understood situation, select the one they think best represents the situation, and explain why (compare two sketch graphs without detailed scales on axes, each of which purport to show how far a person is from home at

Equations and inequalities

various times after leaving school and walking up a hill towards home).

- Sketch graphs that give them a feeling for the relationships in familiar situations without recourse to careful data collection or point plotting (draw 'qualitative' graphs of mood swings during a grand final football match from different points of view).
- Sketch informal graphs to fit verbal descriptions or stories (sketch a graph from a verbal account of noise level during a party).

At level 5, a student:

5.30 Generates numbers or numbers pairs that satisfy a single constraint stated in natural language.

Evident when students, for example:

- Solve number puzzles expressed in words (half a number add 1 is 41, what is the number?).
- Find pairs of numbers that obey a constraint expressed in words (list all the passing combinations of scores if the total on the two parts of the test, each marked out of 10, must be more than 11).
- Plot points to represent constraints (plot points in three colours to represent the lengths and widths of rectangles made from exactly, more than, and less than 36 squares).
- Restate a constraint in a form that clarifies it and makes it easier to solve (needing to find how long the side of a square will have to be if its area must be 1391 cm^2, restate as 'Find a number that multiplies by itself to give 1391').
- Find missing quantities, using strategies that include guess, check and improve (use a four-function calculator to try numbers to see if they square to 1391, attempting to improve the guess each time).
- Describe conditions under which an arithmetic statement will be true or not true (when is it true that dividing one number by another number gives an answer bigger than the first number?).

Level 6 outcomes:

6.28 Uses basic conventions of algebraic notation in representing situations involving a variable quantity and explains why two linear expressions are equivalent.

6.29a Plots, sketches and interprets graphs, considering points, intervals lengths, increases and decreases over an interval, and slope.

6.29b Recognises and represents at least linear and square relationships in tables, symbols and graphs and informally describes how one quantity varies with the other.

6.30 Sets up equations to represent one constraint in a situation, solves equations of the form $ax \pm b = cx \pm d$ and $ax^2 \pm b = c$ using guess, check and improve and graphical methods, and solves linear equations using analytic methods.

LEVEL SIX

LEVEL 6 Statement

Students at level six use a variety of tools and techniques to make accurate drawings. They draw polygons to specifications and create and copy designs based on polygons and circles. They understand and use basic properties of triangles and rectangles and the conditions for congruent triangles. They make and check generalisations about shapes, visualise and sketch the effect of transformations described in geometric language, and describe the basic properties of these transformations. They can visualise and sketch paths and regions given a set of spatial constraints expressed in ordinary language .

WORKING MATHEMATICALLY

SPACE

Students at this level interpret whole powers, roots and scientific notation. They use negative numbers and ratios to describe the relationship between quantities. Their understanding of patterns in the number system and of relationships between the four operations gives them the flexibility to calculate efficiently. They have developed a feeling for common fractions and percentages and use this to make estimates. They know common equivalences and can use these to calculate percentages mentally, either exactly or approximately.

NUMBER

The description for this strand extends over 4 pages

Students are now more independent when measuring or solving measurement problems. If required, they can estimate, reject unreasonable estimates and decide the level of accuracy required. They decide what measurements are needed and how to collect them. They extract measurements from published material and choose and use a wide range of formulae to calculate areas and volumes of parallelograms, triangles, circles, prisms and pyramids, substituting correctly and expressing in appropriate units. They can work with similarity and Pythagoras's theorem to calculate sides of triangles.

MEASUREMENT

At this level, students use systematic strategies to work out probabilities. They use primary or secondary data to assign probabilities for one- and two-stage events. They continue to work individually and collaboratively in planning primary and secondary data collection, taking care in formulating questions and thinking about how data could be organised before it is collected. They use appropriate graphs and summary statistics to represent data. They are able to draw lines of good fit by eye and interpret and draw conclusions from both their work and published data.

CHANCE AND DATA

Students use the basic conventions of algebra to represent variables with letters. Algebraic symbols are used to summarise rules, and students manipulate expressions to explain why two linear expressions are equivalent. They produce and interpret graphs of algebraic relationships, planning their own axes and scales. They distinguish linear from square relationships and recognise these and other types of relationships between variables in tables, symbolic expressions and graphs. Students write symbolic equations and use analytical methods to solve linear and graphical equations and guess, check and improve methods to solve others.

ALGEBRA

LEVEL 6
Table of outcomes

6.1 – 6.6 No outcomes at this level

6.7 Uses geometric techniques and tools to interpret and meet specifications requiring the accurate construction and placement of figures and objects.

See page 100

6.8 Visualises, sketches and describes paths and regions that satisfy conditions given in everyday language.

See page 100

6.9 Analyses, describes and uses relationships in and between classes of figures, including parallel, perpendicular and intersecting lines, triangles and rectangles.

See page 101

6.10 Visualises, produces and accurately describes translations, reflections, rotations and enlargements.

See page 101

6.11 Interprets and uses whole powers and roots, scientific notation, ratios, percentages and negative numbers.

See page 102

6.12 See outcome 6.28

See page 110

6.13 See outcome 6.30

See page 111

6.14 Uses a ratio or familiar rate to describe the relationship between two directly proportional quantities and to calculate one quantity from another.

See page 103

6.15 Estimates and calculates mentally with whole and fractional numbers, including finding frequently used fractions and percentages of amounts.

See page 104

6.16 Uses understood written methods to calculate with decimal and common fractions and integer powers.

See page 104

6.17 Makes efficient use of a scientific calculator, including for powers and roots and using scientific notation.

See page 105

6.18 & 6.19 Decides what measurements are needed to complete a practical task and how to obtain them and make or collect measurements to the planned level of accuracy.

See page 106

6.20 Unprompted, estimates in situations in which it is sensible to do so and judges the reasonableness of estimates.

See page 107

6.21 Integrates information from several sources to determine time and duration of time and to plan and synchronise events.

See page 107

6.22a Understands and uses relationships involving length, area and volume of quadrilaterals and circles, prisms and pyramids.

6.22b Understands and uses similarity and Pythagoras's theorem to solve problems involving right triangles and scale drawing.

See page 107

6.23 Estimates probabilities and proportions based on primary or secondary data collection and assigns probabilities for one- and two-stage events by reasoning about equally likely events.

See page 108

6.24 Plans experiments and surveys, collaboratively and independently deciding how to collect data consistently and the sources and types of data.

See page 108

6.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, planning class intervals and fields collaboratively and individually.

See page 109

6.26 Displays and summarises data to show location and variability (including when some grouping of data is required) to compare data sets and to show relationships in one data set.

See page 109

6.27 Interprets collected and published data from tables, diagrams, plots, graphs, prose and databases to make comparisons and describe relationships.

See page 109

6.28 Uses basic conventions of algebraic notation in representing situations involving a variable quantity and explains why two linear expressions are equivalent.

See page 110

6.29a Plots, sketches and interprets graphs, considering points, interval lengths, increases and decreases over an interval, and slope.

6.29b Recognises and represents at least linear and square relationships in tables, symbols and graphs and informally describes how one quantity varies with the other.

See page 110

6.30 Sets up equations to represent one constraint in a situation, solves equations of the form $ax \pm b = cx \pm d$ and $ax^2 \pm b = c$ using guess, check and improve and graphical methods, and solves linear equations using analytic methods.

See page 111

LEVEL 6 Space

Level 5 outcomes:

5.7a Makes accurate versions of simple mathematical figures and objects, including making own nets.

5.7b Interprets and makes drawings of 3D shapes accurately, using conventions for representing 3D space in 2D with consistency.

5.8 Uses network diagrams to represent the order of, and paths between, locations and events.

5.9 Analyses, describes and uses distinguishing features of common classes of mathematical figures and objects, recognising parallels, perpendiculars and congruence.

5.10 Visualises, moves and sketches shapes to show the effect of translations, reflections, rotations and enlargements (using grids).

Using spatial ideas, tools and techniques to interpret, draw and make

At level 6, a student:

6.7 Uses geometric techniques and tools to interpret and meet specifications requiring the accurate construction and placement of figures and objects.

Evident when students, for example:

- Use a ruler, protractor, pair of compasses and a selection of Mira, Proliner, Geoliner, Rotagram, Math-o-matt and Mathaid to draw parallels and perpendiculars and to copy lines and angles.
- Produce an isometric, oblique or perspective drawing of an object, given a plan view and enough other information, or the reverse.
- Make a model based on specifications provided in isometric, oblique or perspective drawings or plan.
- Use drawing equipment or computer drawing software to copy and design logos and other patterns.
- Construct figures, including triangles, quadrilaterals, circles and ellipses, for which a diagram or dimensions, or both, have been provided (construct a regular hexagon).
- Use mathematical properties to check the accuracy of constructed figures (check the lengths of diagonals of a rectangle).
- Select a suitable means of enlarging and reducing a figure (a projection or grid where a figure is on paper, measurements of sides and angles where a scale drawing or model of a larger thing is needed).

Visualising, analysing and representing arrangements and locations

At level 6, a student:

6.8 Visualises, sketches and describes paths and regions that satisfy conditions given in everyday language.

Evident when students, for example:

- Visualise and draw paths and regions described in everyday language (the region mowed, if constrained by a given length of electric cord and two fixed power sources; the set of all possible locations for a sportsground to be the same distance from each of two competing country towns or three country towns).
- Visualise and sketch solids generated by the translation and rotation of lines, curves and regions (as in a potter's wheel).
- Make generalisations about paths as a result of experimenting with drawing tools (find a way of using a pair of compasses and straight edge to draw a perpendicular bisector to a given line when not previously taught a method).
- Observe, sketch and describe paths of points that move in regular repeating motion (the paths traced out by the valve on a bicycle wheel or the axle ends on a stair-climbing furniture trolley).
- Interpret and use spatial terms such as 'equidistant' and 'bisect' when drawing and describing paths and regions.

Visualising, analysing and representing shapes

At level 6, a student:

6.9 Analyses, describes and uses relationships in and between classes of figures, including parallel, perpendicular and intersecting lines, triangles and rectangles.

Evident when students, for example:

- Test and revise descriptions of types of triangles and rectangles by searching for counter-examples (revise 'polygons with four equal sides are squares').
- Identify congruent, supplementary and complementary angles and use angle relationships in intersecting, parallel and perpendicular lines and triangles to find other angles.
- Design practical checks for whether lines and surfaces are parallel, perpendicular, vertical and horizontal (recognise that it would not be practical to use a protractor to ensure that two walls are parallel, but the fact that the diagonals of a rectangle are the same length and bisect each could be used to determine this).
- Report on shapes that can be produced according to a geometric constraint (a quadrilateral with two sides congruent; a triangle with two sides of 5 cm and 7 cm and the angle between them 60°).
- Use congruence conditions for triangles to solve mathematical and practical problems (respond to 'In these two triangles, I know that this side equals this side — what else do I need to know to say that the two triangles are congruent?').
- Draw on the properties of triangles and rectangles to report on the relationship between shape, structure and function in some built things (differences in roof design in different climatic regions, packaging for display, strength and storage).

Visualising, analysing and representing movements and transformations

At level 6, a student:

6.10 Visualises, produces and accurately describes translations, reflections, rotations and enlargements.

Evident when students, for example:

- Accurately carry out a specified translation, rotation or reflection using coordinate, tracing paper, or geometric drawing equipment (rotate triangle ABC about B, 60° clockwise).
- Describe a reflection, translation or rotation of a figure unambiguously (given a coordinate grid and with a figure on it, transform it and describe the transformation so a partner can reproduce it unseen — 'I reflected in the line through (0, 3) and (4, 0)').
- Produce simple arguments about tessellations, symmetry and transformations (all triangles tessellate since a triangle and its reflection form a parallelogram and all parallelograms tessellate).
- Visualise the effect of a translation, rotation or reflection on the position and orientation of a 3D object (given a diagram of a model, match it to a sketch that shows what would be seen from a different direction).
- Predict how many squares or cubes will be needed to produce a shape enlarged by a small whole number factor (a model made with five cubes is enlarged by a scale factor of 4 — how many cubes will be needed?).
- Enlarge and reduce figures using projections and explain the effect the scale factor has on the lengths and areas of the image.
- Find the centres, axes and planes of symmetry in figures and objects.

Level 7 outcomes:

7.7 Draws on properties of shapes and transformations to plan how to meet specifications requiring the accurate construction or placement of figures and objects.

7.8 Visualises, constructs and describes paths and regions using conventional geometric language, including that based on coordinates.

7.9 Investigates and uses relationships in and between classes of figures, including quadrilaterals and circles.

7.10 Analyses translations, reflections, rotations and enlargements and relates their properties to similarity and congruence.

LEVEL 6 Number

Level 5 outcomes:

5.11 Interprets and uses whole powers and square roots and straightforward ratios and percentages.

5.12 (See outcome 5.28)

5.13 (See outcome 5.30)

5.14 Chooses and sequences several operations, covering situations involving decimal multipliers and divisors and division of smaller by larger numbers.

5.15 Estimates and calculates mentally with whole numbers, money and simple fractions, including multiplying and dividing some two-digit numbers by one-digit numbers.

5.16 Uses understood written methods to add, subtract, multiply and divide whole numbers and common and decimal fractions (whole number multipliers and divisors).

5.17 Makes efficient use of a calculator for common and decimal fractions and percentages and takes into account orders of operations.

Count and order

At level 6, a student:

6.11 Interprets and uses whole powers and roots, scientific notation, ratios, percentages and negative numbers.

Evident when students, for example:

- Interpret and use ratios of the form 'a parts to b parts' when making comparisons of parts within a whole (order ratios —2:5, 3:7, 2:3— for concentrations of mixtures and understand why the mixtures go from weakest to strongest).
- Contrast differences and ratios as means of comparing things (comment sensibly on the effect of adding a cup of water to a litre jug of cordial and to a 9-litre bucket of the same cordial).
- Interpret published ratios and rates to make comparisons (use mortality or illness rates in discussing health issues).
- Use negative numbers to compare and order measures (compare and order minimum temperatures around the world, draw a graph in which 0 is used to indicate the time at which measurements began and negative numbers earlier times).
- Use powers of ten to make order of magnitude comparisons (to compare the sizes of animal species or distances to planets).
- Use scientific notation —also called standard form— to represent very large and very small numbers in manageable form.
- Critically interpret published percentages by analysing the whole to which they refer (state what 'increased by 200%' means and determine whether it is correctly used in an advertisement).

Number patterns

6.12 See outcome 6.28 in Algebra.

Equations

6.13 **See outcome 6.30 in Algebra.**

Applying numbers

At level 6, a student:

6.14 Uses a ratio or familiar rate to describe the relationship between two directly proportional quantities and to calculate one quantity from another.

Evident when students, for example:

- Deal in routine ways with situations involving rates such as price, speed, scale and simple interest (convert actual distances to map distances with a scale ratio of 0.15, calculate interest for six months at 11% per annum).
- Obtain measurements from rates (calculate the number of worm tablets to give a dog, based on its weight and the information on the label).
- Use reference points to estimate the scale factor — the ratio— between an original object and its image (use an estimate of a woman's height to estimate the scale factor of a photograph she is in).
- Recognise direct proportion in familiar situations (at a fixed rate of pay, income will be directly proportional to hours worked).
- Identify the assumptions needed to use a ratio (what do we need to assume to calculate the quantities needed to triple a recipe and would the same assumptions work for cooking time?).
- Use a method, such as the unitary method, to solve problems involving proportional quantities (find the price of 4.5 kg of fruit given the price of 3 kg by first calculating the price of 1 kg).
- Link the expressions 'direct proportion', 'directly related' and 'direct variation' and distinguish situations involving partial variation (rate plus fixed quantity) from those involving direct proportion (rate).
- Increase or decrease a quantity in a given ratio or by a given percentage (convert a recipe for 6 to one for 10).
- Divide a quantity in a given ratio (share a kilogram pack of seed between two areas to be planted, one of 20 m^2 and the other 30 m^2).

LEVEL 6 Number

Mental computation

At level 6, a student:

6.15 Estimates and calculates mentally with whole and fractional numbers, including finding frequently used fractions and percentages of amounts.

Evident when students, for example:

- Mentally add and subtract fractions involving well known equivalences ($2\frac{3}{4} - 1\frac{1}{2}$, $\frac{3}{4} + 1\frac{1}{2}$).
- Calculate percentages of amounts of money or measures to a suitable level of accuracy, including using common fraction equivalents (calculate 50%, $33\frac{1}{3}$%, 25%, 10% and 5% of $45 to the nearest cent).
- Use strategies such as rounding or decomposition to estimate percentages and decimal fractions (as 9% is a bit less than 10%, estimate 15% of an amount by finding 10% and adding half again).
- Explain the effect of different methods of approximation on the closeness of the estimate for different operations.
- Make decisions about whether to round up or down or to the nearest, based on the purpose of the estimation ('For three pairs of socks at $3.66, I'll take $12 rather than $9').

Written computation

At level 6, a student:

6.16 Uses understood written methods to calculate with decimal and common fractions and integer powers.

Evident when students, for example:

- Use understood written methods to add, subtract, multiply and divide decimal fractions, rounding to specified numbers of places (find 13.6 x 11.3 and 95.5 ÷ 0.8 accurate to one decimal place).
- Use understood written methods to add, subtract, multiply and divide common fractions they cannot do completely mentally (write $\frac{3}{4} \times \frac{8}{9} = 3 \times \frac{1}{4} \times \frac{8}{9} = 3 \times \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$; $4\frac{2}{3}$ of $60 = 240 + \frac{2}{3}$ of $60 = 240 + 40 = 280$; $3\frac{1}{2} \div \frac{3}{4} = \frac{7}{2} \div \frac{3}{4} = \frac{14}{4} \div \frac{3}{4} = \frac{14}{3} = 4\frac{2}{3}$).
- Evaluate integer powers ($2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$) and find the product and quotient of numbers expressed as integer powers ($2^5 \times 2^3 = 2^8$, $2^7 \div 2^3 = 2^4$, $2^3 \div 2^8 = 2^{-5}$).
- Use their knowledge of operations on powers to determine the magnitude of a product for numbers expressed in scientific notation (to evaluate $(1.3 \times 10^5) \times (3.7 \times 10^{-4})$).
- Understand for which multipliers, multiplication makes bigger or smaller and for which divisors, division makes bigger or smaller and use this to help check the reasonableness of results.

Calculators

At level 6, a student:

6.17 Makes efficient use of a scientific calculator, including for powers and roots and using scientific notation.

Evident when students, for example:

- Enter and read scientific notation (or standard form) in scientific calculators (read displays 1.456.....02 or 1.436^2 respectively as 1.456×10^2).
- Recognise that different scientific calculators may read the same sequence of key strokes differently, read instruction booklets for calculators and take their particular features into account when entering calculations (to enter x/yz).
- Use an efficient method for evaluating powers and roots on their calculator (use the x^y and $n\sqrt{y}$ keys).
- Use the memory or bracket facilities of a calculator to assist in evaluating complex expressions, like $\sqrt{(1.6^2 + 3.1^2)}$.
- Understand that truncating or rounding calculator displays can affect results of calculator computations.

Level 7 outcomes:

7.11 Selects a suitable form of number representation, explains choice and moves freely between representations.

7.12 (See outcome 7.28)

7.13 (See outcome 7.30)

7.14 Understands the nature of a rate, and chooses, calculates and compares with ratios and rates, including situations involving direct and indirect proportion.

7.15–7.17 Undertakes efficient computations on positive and negative numbers of any size, including rearranging formulas and quoting results to a suitable level of accuracy.

LEVEL 6 Measurement

Level 5 outcomes:

5.18 Takes purpose and practicality into account when selecting attributes, units and instruments for measuring things.

5.19 Measures and makes things, using a range of graduated scales and strategies for making measurements that are more accurate than the available equipment allows.

5.20 Makes sensible estimates of length, area, mass and capacity in common standard units and identifies unreasonable estimates of things.

5.21 Estimates, measures and calculates time and duration of time and uses timelines and a range of types of timetable.

5.22 Understands and uses relationships involving length, area and volume, including those for shapes based on rectangles, triangles and rectangular prisms.

Choosing units Measuring

At level 6, a student:

6.18 & 6.19 Decides what measurements are needed to complete a practical task and how to obtain them and make or collect measurements to the planned level of accuracy.

Evident when students, for example:

- Understand that the choice of unit determines the level of accuracy of a measurement and is actually a decision about how much measurement error is tolerable.
- Decide what variables need to be measured, directly or indirectly, to complete a practical task (preparing an interior decorating plan for the school's reception area, or determining minimum food needs for survival).
- Identify and gather the information needed to estimate quantities that cannot be measured conveniently (the number of trees used to publish a Saturday edition of The Australian newspaper).
- Select and use collected, provided and derived measurement data to solve a practical problem (use occupational health and safety data when judging the suitability of their computer work station).
- Select the same unit to measure things for computational purposes (when calculating average mass, select or convert all measurements to grams).
- Investigate and report on methods of measuring attributes to a high level of accuracy (electronic timing of Olympic swimming records).

Estimating

At level 6, a student:

6.20 Unprompted, estimates in situations in which it is sensible to do so and judges the reasonableness of estimates.

Evident when students, for example:

- Elect to estimate rather than accurately measure in situations where estimation is a sensible or convenient approach.
- Judge the adequacy of an estimate in terms of its purpose, rather than its exactness (since the paint comes in 1- or 4-litre cans and four litres costs about the same as two 1-litre cans, getting an estimate of 'about one litre' is unhelpful).
- Work collaboratively to pool information and estimate quantities they cannot measure directly or conveniently (the number of babies born in Australia each day, how many apples are eaten in their community each year).
- Investigate methods of estimating long time periods from natural phenomena (age of trees from growth rings, carbon dating).
- Decide when it is better to under-estimate and when to over-estimate (when there is no possibility of buying more of the same tiles later, an over-estimate of the number needed is sensible).
- Explain the basis of their estimates and justify strategies used.

Time

At level 6, a student:

6.21 Integrates information from several sources to determine time and duration of time and to plan and synchronise events.

Evident when students, for example:

- Determine the time in each state of Australia given the time in one state and information about daylight saving.
- Calculate elapsed time from a timetable, taking into consideration time zones and daylight saving (compare the time taken to fly from Perth to Sydney and from Sydney to Perth).
- Synchronise practical activities that must all be ready at the same time (plan meal preparation, a dance).
- Produce travel schedules by integrating information from several sources (produce travel plans for people arriving from all over Australia for a meeting of the Aboriginal and Torres Strait Islander Commission, integrating such elements as flight times, transport from airports).
- Use time as a measure of distance, both informally ('I live 30 minutes from town') and formally (light years).
- Use time sheets, definitions of such things as 'overtime' and 'double time', wage rates and PAYE to calculate wages based on hours worked.

Using relationships

At level 6, a student:

6.22a Understands and uses relationships involving length, area and volume of quadrilaterals and circles, prisms and pyramids.

Evident when students, for example:

- Understand that all parallelograms on the same base and of the same height will have the same area, as will all triangles of the same base and height.
- Select suitable formulae and measurements to calculate the area of rectangles, triangles, parallelograms, trapezia and circles and the surface area and volume of prisms.
- Collect data on the volume of related shapes and make suitable generalisations (investigate the base area and volume of prisms of the same height and conjecture that the volume is proportional to the area of the base).
- Use knowledge that the volume of a prism is the area of the base times the vertical height and that the volume of the analogous pyramid is one-third of the volume of the prism to solve problems involving slant and upright prisms and pyramids with a variety of base shapes.
- Apply commonly used formulae correctly, that is, substitute known values correctly, use suitable units and interpret the solution (for finding the surface area and volume of prisms and pyramids).
- Relate area and volume units to length units and make associated conversions (convert square metres to square centimetres).

6.22b Understands and uses similarity and Pythagoras's theorem to solve problems involving right triangles and scale drawing.

Evident when students, for example:

- Use Pythagoras's theorem to find lengths of sides of right triangles (find the distance above the ground of a kite; given the string length and horizontal distance, find the length of the side of a square inscribed in a circle of given radius).
- Produce scale drawings of given figures using rays or some other method (make a copy of this quadrilateral to one-third size).
- Produce scale drawings of figures based on information about the dimensions and angles of the object (the side view of a window fitting from information in a plan).
- Select a suitable scale factor to achieve results (to ensure their scale drawing fits on the page or that their linear scale will give the expected 3D size).
- Use the ratio rise/run to solve problems involving heights and distances.
- Take appropriate measurements —lengths and angles— and use similarity relationships between right triangles to find distances and heights in inaccessible situations (to draw a map or plan).

Level 7 outcomes:

7.18 No outcome at this level

7.19 No outcome at this level

7.20 Appreciates that all measurements involve error and estimates the extent of uncertainty in direct and indirect measures.

7.21 No outcome at this level

7.22a Selects and uses, both directly and indirectly, formulae for length, area and volume of figures and objects (including for spheres).

7.22b Understands and uses similarity relationships in and between figures and objects, including with trigonometric ratios.

LEVEL 6 Chance and Data

Level 5 outcomes:

5.23 Interprets and makes numerical statements of probability based on lists of equally likely outcomes and using fractions and percentages.

5.24 Collaborates in planning and refining survey questions and observation methods for collecting frequency and measurement data.

5.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, using class intervals and fields provided or planned with help.

5.26 Displays one-variable and two-variable data in plots and summarises data with fractions, percentages, means and medians.

5.27 Reads and describes information in histograms, plots and summary statistics and reports on data collection processes and results.

Understanding, estimating and measuring chance variation

At level 6, a student:

6.23 Estimates probabilities and proportions based on primary or secondary data collection and assigns probabilities for one- and two-stage events by reasoning about equally likely events.

Evident when students, for example:

- Use published data to assign probabilities to events (use knowledge that on average it rains on 40% of days in July in a particular place to assign a probability of 0.4 for a day in July being wet).
- Estimate proportions from experimental data (use draws, with replacement, of differently coloured beads from a bag to estimate the proportion, or number, of each colour bead in the bag; use data collected on cars travelling along the highway to estimate the proportion of red cars in the district).
- Estimate probabilities for compound events based on information organised in tables, diagrams and graphs (use census data organised into a nested table to estimate the probability of an Australian family having a female as sole 'head of household').
- Assign probabilities to compound events based on tree diagrams, two-way tables and systematic lists (to solve 'If a pair each of red socks, white socks and blue socks were thrown in the drawer unmatched, what is the probability that two socks grabbed in the dark will match?').
- Use census data to assign probabilities (use data about their school population to decide the probability that a member of a school club will be an Aborigine).

Collecting data

At level 6, a student:

6.24 Plans experiments and surveys, collaboratively and independently, deciding how to collect data consistently and the sources and types of data.

Evident when students, for example:

- Plan and carry out simple experiments and surveys to estimate proportions, including those based on a simulation with teacher guidance in designing the simulation itself (plan to draw balls, with replacement, to estimate the proportion of each colour in a bag; plan a survey of eye colour to estimate the proportion of people with brown eyes).
- Explain what a simulation is in relation to an actual experiment (explain that we can toss a coin or coins to simulate families of four children rather than having to have 100 families).
- Collect information from a variety of sources including databases and give reasons for their choice (from a telephone book, classified advertisements, dictionary, class list, computer database, kinship diagram, regional, state or national census data).
- Plan ways to ensure that data is collected consistently (in considering 'Do students do more homework as the year goes on?', decide what is meant by 'doing homework' and if 'how much homework' means time spent or some other measure).
- Distinguish between a sample and the population where the distinction is clear (describe the sample as the year 9 students questioned and the population as all year 9 students in the school).

Organising data

At level 6, a student:

6.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, planning class intervals and fields collaboratively and individually.

Evident when students, for example:

- Organise raw data to examine it for errors and inconsistencies (group data on pulse rates after strenuous exercise, identify two outliers and conclude they were possibly due to recording errors).
- With prompting, plan class intervals with a sensible number of intervals for the purpose and covering the range for continuous data (250 mL $\leq$ v < 500 mL, 500 mL $\leq$ v < 750 mL, 750 mL $\leq$ v < 1000 mL...).
- Organise data in nested tables planned collaboratively or with guidance (organise data relating to preferred sport by nesting gender under age range).
- Collaboratively, or with guidance, identify variables that help answer questions (to investigate gender and advertising, identify variables such as product, who the advertisement was directed at, gender of the voice-over, tone of voice).
- Set up fields for developing a simple database based on previously identified variables (to develop a database of advertisements).
- Enter and manipulate data in a spreadsheet with a template already set up (in investigating the amount of water and organic matter in various soil samples, enter data on wet, dried and burned weights of soil samples in a spreadsheet).

Displaying and summarising data

At level 6, a student:

6.26 Displays and summarises data to show location and variability (including when some grouping of data is required) to compare data sets and to show relationships in one data set.

Evident when students, for example:

- Represent grouped univariate data in histograms (equal interval width).
- Use methods such as back-to-back stem plots and bar graphs and double column bar graphs to compare two sets of univariate data (use a back-to-back stem plot to compare age distributions of males and females).
- Use fractions and percentages to describe the variability (or spread or dispersion) of results (all the estimates were between 21 and 35 seconds, but 80% were between 27 and 31 seconds).
- Represent bivariate data in scatter plots and, where appropriate, sketch lines of good fit 'by eye' (to investigate relationships between food and drink purchases at the canteen and the weather).
- Represent bivariate time series data in line graphs (to investigate the pattern of homework over a period).
- Understand and use weighted means (class A has 30 students and a mean score of 15, Class B has 25 students and a class mean of 18. What is the mean for all 55 students?).

Interpreting data

At level 6, a student:

6.27 Interprets collected and published data from tables, diagrams, plots, graphs, prose and databases to make comparisons and describe relationships.

Evident when students, for example:

- Explain orally or in writing what is indicated by summary statistics, tables and graphs taken from sources such as magazines, newspapers, information leaflets, school textbooks.
- Consider largest and smallest values, outliers, gaps, clusters, and the relative position of items of importance in interpreting plots.
- Report on what their own displays and summary statistics show about similarities and differences between two data sets (compare hours spent in household work by girls and boys).
- Informally draw inferences from time series data and predictions from trend lines (even though each day is slightly different, the pattern of the tides is quite similar every day).
- Make predictions based on samples (take samples from a bag of green and red balls and make predictions such as 'I think that there are about twice as many red balls as green balls in the bag').
- Interpret data from databases including that requiring the logical connectors 'and', 'or' and 'not' (these students are taller than 155 cm and lighter than 40 kg. They are probably underweight for their height.).

Level 7 outcomes:

7.23 Estimates probabilities, proportions, means and medians based on primary and secondary data collection and assigns probabilities using complementarity and independence.

7.24 Plans experiments, simulations and surveys, collaboratively and independently, considering the appropriateness and quality of observations and the suitability of samples.

7.25 Compares, chooses and uses methods of organisation to suit the type of data and the questions asked.

7.26 Displays and summarises data to show location, variability and association, and links displayed data with measures of location, variability and association.

7.27 Selects and interprets information from collected and published data to construct arguments.

LEVEL 6 Algebra

Level 5 outcomes:

5.28 Follows and constructs rules for describing sequences and relationships between variable quantities using mostly natural language.

5.29a Generates and plots data in first quadrant coordinate graphs, describing patterns in the resulting scatter of points.

5.29b Informally sketches and interprets graphs that describe the relationship between two quantities in everyday situations.

5.30 Generates numbers or number pairs that satisfy a single constraint stated in natural language.

Expressing generality

At level 6, a student:

6.28 Uses basic conventions of algebraic notation in representing situations involving a variable quantity and explains why two linear expressions are equivalent.

Evident when students, for example:

- Use algebraic expressions to represent some aspect of reality or imagination (write weekly expenditure on bus fares as $10x - 1.5$ where x is the face value of a ticket and $1.50 the discount on a book of 10 tickets).
- Translate simple linguistic statements into symbolic statements using algebraic conventions such as $y + y + y + y = 4y$, $w \times w \times w = w^3$, $6 \times a = a \times 6 = 6a$, $a \times b = ab = a.b$ (write 'the number seven less than n' as $n - 7$; translate 'I bought a watch and then I bought three more at the same price' to $t = p + 3p = 4p$).
- Explain differences between expressions such as $4x + y$ and $4(x + y)$ (by relating to the ambiguity in linguistic expressions such as '4 apples and bananas' and by substituting numerical values).
- Understand and use the distributive property for a product of a whole number with a linear expression (use an array to demonstrate that $2(b + 4) = 2b + 8$).
- Use materials or diagrams to explain why two expressions are equivalent for whole number values of n (build the same sequence of shapes in two ways to show $5n + 1 = 6n - (n - 1)$).
- Use a realistic situation to explain why two expressions are equivalent (If m was the number of years old Mum is and a the number of years old I am, then $m = 2a + 3$ means Mum is twice my age plus another three years. So if Mum was three years younger she would be twice my age and this can be written as $m - 3 = 2a$.).
- Use linear expressions to make general arguments (describe three consecutive numbers as $n - 1$, n and $n + 1$ and show that the sum of three consecutive numbers will be a multiple of three).

Function

At level 6, a student:

6.29a Plots, sketches and interprets graphs, considering points, interval lengths, increases and decreases over an interval, and slope.

Evident when students, for example:

- Follow symbolic rules to generate input/output pairs and draw graphs (from formulae or the function keys of a calculator).
- Consider the range of values to be fitted on each axis when selecting scales for graphs (for a conversion graph, use the range of values of one variable to estimate the range of values of the other).
- Interpret local features of graphs such as points and intervals (read tide charts for heights and height changes over a time range).
- Relate slope to the rate at which one quantity varies with another (in distance-time graphs for jogging at different steady paces).
- Understand how a graph shows one variable increasing or decreasing with another (compare a positively sloped graph of growth in average youth wages over time with a negatively sloped graph of number of weeks taken to earn $5000 at those wages).
- Sketch graphs that show steady, increasing or decreasing change of one variable with another (water level over time in jars with vertical or sloped sides as water drips in at a steady rate).
- Draw lines of good fit 'by eye' and use to predict new values where appropriate (fit a line to data on babies' average weights for the first 12 weeks, note that a line fits 'quite well' but extrapolation beyond a few months produces obviously wrong values).

6.29b Recognises and represents at least linear and square relationships in tables, symbols and graphs and informally describes how one quantity varies with the other.

Evident when students, for example:

- Describe, in symbols, rules that are linear or involve square expressions (represent the general term for the variable 'number of matches' in a matchstick pattern with an expression such as $5n - 1$ or $(n - 1)^2$).
- Look for difference patterns in tables of data to decide whether a sequence could be linear or square.
- Understand how the ideas of proportion, direct variation, partial variation, steady change and linear relationship are linked (compare charges based on an hourly rate and those based on a fixed charge plus hourly rate).

Equations and inequalities

- Informally describe how two quantities in simple formulae vary with each other (for $F = 4t$, state that as t doubles (or triples) so too does F; but for $F = 4t^2$ as t doubles F quadruples).
- Match graphs with particular linear relationships by inspecting the constants (choose graphs to match x, 2x, $2x + 3$, $x + 3$, $3 - x$).
- Relate constants a and b in $y = ax + b$ to the contexts from which the variables come (compare rules relating distance and time for various speeds and starting points).
- Distinguish linear, square, cubic and exponential forms (compare sequences and graphs from 2x, x^2 and 2^x; classify 4s, 12s, s^2, $6s^2$, s^3 relating them to attributes of a cube of side lengths).

At level 6, a student:

6.30 Sets up equations to represent one constraint in a situation, solves equations of the form $ax \pm b = cx \pm d$ and $ax^2 \pm b = c$ using guess, check and improve and graphical methods, and solves linear equations using analytic methods.

Evident when students, for example:

- Use algebraic symbols to write equations with one or two variables from a description of a single constraint (write 'seven more than the number is double the number add 1' as $n + 7 = 2n + 1$, 'the rectangle is twice as long as it is wide' as $L = 2W$).
- Relate equations to verbal descriptions of situation (select which of the provided 'stories' matches the equation $x^2 + 1 = 101$; provide an appropriate story about sharing prize money among team members to fit $5x + 40 = 100$).
- Select from 'backtracking' —reverse flow charts— and 'do the same thing to both sides' to solve linear equations such as $4r + 7 = 35$ and $3m + 5 = 37 - m$.
- Use graphing and numerical guess, check and improve for solving, consistent with their algebraic representation skills, a range of equations describing the constraints in a situation.
- Validate solutions to equations by substitution and decide whether a suggested value could be a solution to an equation by substitution rather than by solving the equation.
- Understand that there may be no, one, or more than one, number fitting a constraint (use a graph of $x^2 + 2 = y$ to show for what values of y there are no, one or more than one value(s) of x).

Level 7 outcomes:

7.28 Uses algebraic notation in representing general properties of numbers and relationships between variables and establishes equivalence, including by using the distributive property and inverses of addition and multiplication.

7.29a Plots, sketches and interprets graphs in four quadrants, considering local and global features, including maxima and minima and cyclical changes.

7.29b Recognises and represents at least linear, reciprocal, exponential and quadratic functions in tables, symbols and graphs and describes assumptions needed to use these functions as models.

7.30a Sets up equations and pairs of simultaneous equations to represent constraints in a situation and solves, using guess, check and improve, graphical and, for linear equations, analytic methods.

7.30b Sets up inequalities to represent one or two constraints in a situation and generates a complete set of numbers or number pairs that satisfy the constraints.

LEVEL SEVEN

LEVEL 7 Statement

Students at level seven are able to describe key features of figures. They have developed some understanding of how these properties can be deduced from other properties and can produce a short chain of arguments to establish results. They use their knowledge of the properties of shapes and transformations to plan the construction and placement of figures and objects according to specifications. They solve problems involving congruence and similarity on either Euclidean results or transformations.

At this level, students show number sense, choosing a form of representation sensible for the task at hand. They use ratios to describe the relationship between quantities that are directly or inversely proportional and solve associated problems. Students use their understanding of what a rate is to make sense of an unfamiliar situation. They monitor their own computational work, making decisions about accuracy, checking their results and finding sources of errors. When estimating, they make decisions about how to round based on the purpose of the estimate and the level of accuracy of given information.

Level seven students use a range of standard formulae in indirect ways to calculate lengths, areas and volumes of two- and three-dimensional shapes. They use similarity and trigonometric ratios to solve problems involving at least right triangles and understand the effect of scaling linear dimensions on the areas and volumes of shapes. They understand that measurement involves error and can make sensible statements about acceptable levels of error.

Students understand the difference between a sample and a population in straightforward situations. They take note of sample size and method of collection when planning data collection and making generalisations. They identify faults in data collection instruments, attempt to improve them, and can run simulations to estimate population means, medians and probabilities. They assign probabilities by analysis of the situation and work with some real data sets, using both primary and secondary data, make sensible statements about organising and representing the data, and analyse summary statistics.

At this level, students use algebraic notation to represent polynomial, reciprocal and exponential expressions. They manipulate expressions independently of their context and are fluent in manipulating products, factorisation for simplification purposes. They recognise and sketch families of functions in tables and graphs such as linear, quadratic, exponential and reciprocal. They formulate and solve equations related to these functions, using simple numerical or graphical means as well as analytical methods to solve linear, simultaneous and quadratic equations.

Students use their mathematics in the solution of a range of problems. They pose, clarify and refine questions to help them understand a problem or to guide their investigations. They identify assumptions they need to make when choosing a particular approach and check that their answers make sense. They make and test generalisations and use conventional forms when reading, writing and speaking mathematically. They are beginning to appreciate that the development and uses of mathematics can be linked to the conditions and concerns of the people and communities that produced them.

WORKING MATHEMATICALLY

SPACE

NUMBER

MEASUREMENT

CHANCE AND DATA

ALGEBRA

LEVEL 7
Table of outcomes

7.1 Poses, clarifies and refines mathematical questions to help understand or guide the investigation of a situation. See page 116	7.2 Makes generalisations by abstracting common mathematical features from situations, tests with additional cases and explains why generalisations must be true. See page 116	7.3 Uses problem-solving strategies that include identifying and working on related problems or sub-problems. See page 116	7.4 Applies standard methods or models, comparing and choosing between alternatives, including by considering assumptions needed and results obtained. See page 117	7.5 Uses conventional mathematical language to help give clear and logical accounts of mathematical work. See page 117	7.6 Makes links between the development and use of mathematical ideas and the conditions and concerns of the individuals and communities that produce them. See page 117
7.7 Draws on properties of shapes and transformations to plan how to meet specifications requiring the accurate construction or placement of figures and objects. See page 118		7.8 Visualises, constructs and describes paths and regions using conventional geometric language, including that based on coordinates. See page 118	7.9 Investigates and uses relationships in and between classes of figures, including quadrilaterals and circles. See page 119	7.10 Analyses translations, reflections, rotations and enlargements and relates their properties to similarity and congruence. See page 119	
7.11 Selects a suitable form of number representation, explains choice and moves freely between representations. See page 120	7.12 See outcome 7.28 See page 126	7.13 See outcome 7.30 See page 127	7.14 Understands the nature of a rate, and chooses, calculates and compares with ratios and rates, including situations involving direct and indirect proportion. See page 120	7.15-7.17 Undertakes efficient computations on positive and negative numbers of any size, including rearranging formulas and quoting results to a suitable level of accuracy. See page 121	
7. 18 & 7.19 No outcomes at this level			7.20 Appreciates that all measurements involve error and estimates the extent of uncertainty in direct and indirect measures. See page 123	7.21 No outcome at this level	7.22a Selects and uses, both directly and indirectly, formulae for length, area and volume of figures and objects (including for spheres). 7.22b Understands and uses similarity relationships in and between figures and objects, including with trigonometric ratios. See page 123
7.23 Estimates probabilities, proportions, means and medians based on primary and secondary data collection and assigns probabilities using complementarity and independence. See page 124	7.24 Plans experiments, simulations and surveys, collaboratively and independently, considering the appropriateness and quality of observations and the suitability of samples. See page 124	7.25 Compares, chooses and uses methods of organisation to suit the type of data and the questions asked. See page 125	7.26 Displays and summarises data to show location, variability and association, and links displayed data with measures of location, variability and association. See page 125	7.27 Selects and interprets information from collected and published data to construct arguments. See page 125	
7.28 Uses algebraic notation in representing general properties of numbers and relationships between variables and establishes equivalence, including by using the distributive property and inverses of addition and multiplication. See page 126		7.29a Plots, sketches and interprets graphs in four quadrants, considering local and global features, including maxima and minima and cyclical changes. 7.29b Recognises and represents at least linear, reciprocal, exponential and quadratic functions in tables, symbols and graphs and describes assumptions needed to use these functions as models. See page 126	7.30a Sets up equations and pairs of simultaneous equations to represent constraints in a situation and solves, using guess, check and improve, graphical and, for linear equations, analytic methods. 7.30b Sets up inequalities to represent one or two constraints in a situation and generates a complete set of numbers or number pairs that satisfy the constraints. See page 127		

LEVEL 7 Working Mathematically

Level 6 outcomes:

No outcomes at this level

Level 5 outcomes:

5.1 Begins and extends tasks by asking some mathematical questions, including 'What would happen if...'.

5.2 Understands a conjecture as a guess with reasons and draws on mathematical knowledge to give reasons for conjectures before testing them.

5.3 Uses problem-solving strategies, including those based on selecting and organising key information and being systematic.

5.4 Checks that answers fit specifications and make sense in the original situation.

5.5 Uses mathematical terms and notations with some care to describe objects and relationships and report conclusions with clarity.

5.6 Explains some ways mathematics is used, or has been used in the past, to represent, describe and explain our world.

Investigating

At level 7, a student:

7.1 Poses, clarifies and refines mathematical questions to help understand or guide the investigation of a situation.

Evident when students, for example:

- Choose useful criteria for determining a best option and decide which can and cannot be dealt with mathematically (given 'The local newspaper claims that Darwin residents would be better off flying to Adelaide, buying a car and driving it back than buying it locally — is this so?', they refine 'better off', decide where mathematics can help and plan specific questions to be tackled).
- Ask questions of clarification when faced with ambiguity (when asked to generate pairs whose product is 48, ask whether the numbers must be whole or can be negative or fractional).
- Given a stimulus, such as an unmarked diamond shape, generate questions for investigation (a diamond has two pairs of adjacent sides congruent — what else follows? Is that enough to define it? Will there always be a pair of congruent angles? What else follows if all the sides are congruent?).
- Articulate questions for themselves to guide their research into a mathematical topic (when researching the importance of prime numbers, ask 'Why not 1?' and 'What is so special about numbers with two factors?').
- Articulate questions to guide their research into a practical problem where mathematics is likely to help (to prepare a savings plan to purchase a CD player, list the answers they need about price, options, weekly disposable cash and interest that can be earned).

Conjecturing

At level 7, a student:

7.2 Makes generalisations by abstracting common mathematical features from situations, tests with additional cases and explains why generalisations must be true.

Evident when students, for example:

- Make generalisations based on the common features of situations (investigate the decimal form of proper fractions and generalise 'When the denominator is 9, the decimal has a recurring cycle of one digit, which is the numerator?').
- Check all cases where possible (check the decimal form of all proper fractions with denominator 7; conjecture that more possible combinations of the score on two dice sum to seven than to any other number, test by listing all combinations).
- Use the structure of a situation or pattern to explain why a generalisation must be true (explain a generalisation for a matchstick pattern by describing how each term is built).
- Test definitions or descriptions of mathematical concepts by searching for counter-examples (provide a counter example to the statement, 'Polygons with four equal sides are squares').
- Use algebra to make general arguments (to show that the product of two odd numbers must be odd).
- Explain why a solution to a practical problem must work in general (explain why their method for finding the centre of a circular garden bed will work).

Using problem-solving strategies

At level 7, a student:

7.3 Uses problem-solving strategies that include identifying and working on related problems or sub-problems.

Evident when students, for example:

- Partition a problem into cases and solve each separately (for a problem involving finding the minimum number of toastings needed to toast multiple slices of bread if the toaster toasts two slices but one side at a time, solve separately for an even number of slices and for an odd number of slices).
- Solve a series of special cases and look for something general (to find a rule relating the number sum of the interior angles of a polygon to the number of side, begin by trying some cases and looking for something general).
- Solve a series of simpler versions of a problem and look for patterns that may help with the original problem (for a problem involving exchanging the positions of 6 red and 6 black pegs on a pegboard, develop a strategy by working with 1, 2, 3, 4... pegs, record the minimum number of moves each time, then look for a pattern).
- Determine the sub-tasks involved in undertaking a practical problem (organise the questions to be addressed and tasks to be completed into sensible clusters and assign tasks to group members).
- Work backwards by using the information provided in a problem in the reverse order to that given (in a puzzle such as 'What day precedes the day after tomorrow if four days ago was two days after Wednesday' or in the more practical task of trying to use a six-day timetable).

Applying and verifying

At level 7, a student:

7.4 Applies standard methods or models, comparing and choosing between alternatives, including by considering assumptions needed and results obtained.

Evident when students, for example:

- Recognise situations to which routine techniques are applicable although not immediately or obviously so (see a baseball diamond as a square and the distance from home to second base as the hypotenuse of a right triangle and use the Pythagorean result).
- Use a given model to make predictions and test them against real data (given a linear model in the form, $f(x) = ax + b$, for the number of weeks needed to earn $5000 at average weekly earnings for the period 1976 to 1982, test its predictive power for more recent years).
- Compare different models as predictors (compare models of the forms, $f(x) = A/x$ and $g(x) = -x + B$, as predictors of the relationship between house prices and distance from the city centre).
- Identify the assumptions in some standard methods and make reasonable comments about their validity (what do we need to assume to estimate the time taken to complete a run? Would the same assumptions work for any length run?).
- Explain why a solution is reasonable even if it is not fully accurate or correct ('The playing field isn't a rectangle, it's more like an oval but we wouldn't spread the seed that accurately anyway and the spare bits at the corners will make sure that we don't run out').

Using mathematical language

At level 7, a student:

7.5 Uses conventional mathematical language to help give clear and logical accounts of mathematical work.

Evident when students, for example:

- Interpret prepositions — by, with, from — in mathematical expressions (deciding whether the question 'By how much does the number x exceed the number y' can be answered by the operation of subtracting y from x).
- Explain the meaning of familiar mathematical terms and expressions — factor, parallelogram — redrafting to remove ambiguity.
- Move smoothly between graphical, algebraic and verbal forms of representation.
- Express technical symbolic expressions orally (read aloud $f(x) = 2x + 1$, say $(a + b)^2$ as 'a plus b all squared', rather than 'a plus b squared').
- Read reasonably non-technical mathematics and explain the essential ideas to peers (read about and make an oral and written report on Fibonnaci numbers).
- Read to find out about some mathematics needed to solve a specific problem (read a text that explains how to operate with the indices needed for a problem about a bouncing ball or how to find the volume of a slanted truncated pyramid).

Working in context

At level 7, a student:

7.6 Makes links between the development and use of mathematical ideas and the conditions and concerns of the individuals and communities that produce them.

Evident when students, for example:

- Develop a timeline for the development of a familiar mathematical idea or topic reflecting the contribution of various communities and individuals and considering what might have provoked interest (Pascal's triangle, π, Pythagoras's theorem, magic squares, place value, prime numbers and factoring, fractions, equation-solving).
- Link the development of particular mathematical ideas and methods they are learning to the interests, needs and resources of the communities that contributed to their development (probability and aspects of Orthodox Jewish law such as casting lots fairly, deciding what is Kosher, linear programming and the Berlin airlift).
- Identify instances of mathematical ideas studied for interest rather than for any obvious or immediate utilitarian reason (problems such as the trisection of the angle, tessellations, number theory).
- Identify instances of practical interest in particular mathematics developing long after the basic ideas (the study of factors and primes considered eccentric before changes in technology and Cold War interest in cryptography).
- Explain how some developments in mathematics, its applications, or in school mathematics were unlikely without computing technology (the study of fractals, their own study of functions through a graphics calculator).

Level 8 outcomes:

8.1 Shows persistence, autonomy, flexibility and self-reliance when working mathematically.

8.2 Produces mathematical arguments to convince others of the truth of propositions, including those involving deductions from known information.

8.3 Uses problem-solving strategies that include those based on generalising from one problem situation to another and rethinking problem conditions and constraints.

8.4 Formulates models by making useful and simple assumptions, collecting data needed and representing the relevant relationships in mathematical terms.

8.5 Makes fluent use of mathematical notation in solving problems and presenting arguments succinctly, coherently and in conventional forms.

8.6 Appreciates that there is a relationship between mathematics and social conditions and values, commenting on the role of mathematics in describing and shaping aspects of our lives.

LEVEL 7 Space

Level 6 outcomes:

6.7 Uses geometric techniques and tools to interpret and meet specifications requiring the accurate construction and placement of figures and objects.

6.8 Visualises, sketches and describes paths and regions that satisfy conditions given in everyday language.

6.9 Analyses, describes and uses relationships in and between classes of figures, including parallel, perpendicular and intersecting lines, triangles and rectangles.

6.10 Visualises, produces and accurately describes translations, reflections, rotations and enlargements.

Using spatial ideas, tools and techniques to interpret, draw and make

At level 7, a student:

7.7 Draws on properties of shapes and transformations to plan how to meet specifications requiring the accurate construction or placement of figures and objects.

Evident when students, for example:

- Decide which, and how precisely, positions, lines and angles in a situation should be represented in a diagram and draw and label diagrams to represent this spatial information.
- Develop methods to construct specified quadrilaterals using only compass and rule or Mira mirror (to construct a parallelogram given two adjacent sides).
- Apply properties of chords, tangents and angles in circles to construction problems (use a variety of methods to construct a tangent from a point to a circle, construct the smallest square containing a given circle, find the largest rectangle that can be inscribed in a given circle).
- Compare two construction methods and explain why one is preferred (compare compass and protractor methods for constructing a perpendicular or bisecting an angle).
- Examine a simple drawing instrument and explain why it works (why a particular setting on a pantograph produces the required scale copy of a given drawing).

Visualising, analysing and representing arrangements and locations

At level 7, a student:

7.8 Visualises, constructs and describes paths and regions using conventional geometric language, including that based on coordinates.

Evident when students, for example:

- Construct the locus of an object moving according to a rule expressed in conventional mathematical language (the locus of points equidistant from points A and B, the locus of points such that the sum of the distance from points A and B is 12 cm).
- Design instructions for a computer to produce desired paths and regions (use LOGO to prepare graphic designs).
- Use coordinates, in four quadrants, to specify locations for specific points (given coordinates of three vertices of a parallelogram, state the fourth).
- Interpret and use coordinates in constructing and describing simple loci, such as find a line parallel to $y = 3x - 4$ and passing through (1, 2).
- Fold paper to explain why a path satisfies given conditions (repeatedly fold an edge — the directrix — of a piece of paper along a fixed point, its focus, so that the outline of the folds produces a parabola).
- Investigate and describe the features of various conic sections.

Visualising, analysing and representing shapes

At level 7, a student:

7.9 Investigates and uses relationships in and between classes of figures, including quadrilaterals and circles.

Evident when students, for example:

- Describe the key features of various classes of quadrilaterals and show inclusivity relationships (produce a branching tree showing the relationship between quadrilaterals, trapezia).
- Apply the properties of special quadrilaterals such as rhombuses and parallelograms to solve problems (use that the diagonals of the rhombus bisect each other at right angles to enable the application of Pythagoras's theorem to a problem).
- Produce short chains of traditional Euclidean statements to prove propositions about triangles and quadrilaterals (prove that an angle in a semicircle is a right angle).
- Use conventional language associated with circles (chord, semi-circle, sector, angles subtended by an arc).
- Explain relationships in figures involving circles (explain why an angle in a semicircle is a right angle).
- Explain methods for finding the centre of a circle in terms of the properties of chords and tangents (for compass constructions and paper folding).
- Choose materials and geometric properties of circles to carry out practical tasks (place 10 lights around a circular playing field, make a pattern of pieces for a circular skirt, find the centre of a circular herb garden to place a sundial).

Visualising, analysing and representing movements and transformations

At level 7, a student:

7.10 Analyses translations, reflections, rotations and enlargements and relates their properties to similarity and congruence.

Evident when students, for example:

- Understand that translated, rotated or reflected figures and objects are congruent to the original and apply to mathematical and practical problems (use properties of reflection to show that the base angles of an isosceles triangle are congruent).
- Understand that enlarged or reduced figures and objects are similar and apply to mathematical and practical problems (given the size of an image for a projector a certain distance from a screen, calculate where the projector needs to be placed to produce an image of another size).
- Given a figure and its image use various techniques to identify an appropriate one- or two-stage translation, rotation or reflection (use a Mira to find a reflection line and describe it by naming two coordinates; use perpendicular bisectors of the line joining corresponding points to find a centre of rotation).
- Investigate and describe the basis of familiar technology using the language and properties of transformations and symmetry (kaleidoscope, merry-go-rounds, swings, production lines, projectors, linear and rotating motion in household machines).
- Describe the solution to simple practical problems in terms of transformations (plan and explain the placement of two mirrors so light from a fixed source will be reflected to a particular spot).

Level 8 outcomes:

8.7–8.10 Recognises the systematic nature of a geometry and draws flexibly upon, and sees connections between, results about shapes, transformations and locations.

LEVEL 7 Number

Level 6 outcomes:

6.11 Interprets and uses whole powers and roots, scientific notation, ratios, percentages and negative numbers.

6.12 (See outcome 6.28)

6.13 (See outcome 6.30)

6.14 Uses a ratio or familiar rate to describe the relationship between two directly proportional quantities and to calculate one quantity from another.

6.15 Estimates and calculates mentally with whole and fractional numbers, including finding frequently used fractions and percentages of amounts.

6.16 Uses understood written methods to calculate with decimal and common fractions and integer powers.

6.17 Makes efficient use of a scientific calculator, including for powers and roots and using scientific notation.

Count and order

At level 7, a student:

7.11 Selects a suitable form of number representation, explains choice and moves freely between representations.

Evident when students, for example:

- Select a suitable form of number representation and explain their choice (say, 'For $\sqrt{5} \times \sqrt{7}$ it was quicker to keep them as irrationals, get $\sqrt{35}$, and then use my calculator to find a decimal approximation' or 'Standard form was useful because we wanted to compare very large distances').
- Make rational estimates of irrational numbers including by using guess, check and improve (begin with $\sqrt[3]{6}$ is obviously more than one and less than 2', and test to decide it is just over 1.8).
- Recognise index notation involving fractional indices (interpret $64^{1/2}$ as $\sqrt{64}$).
- Understand that all common fractions can be expressed as a recurring or terminating decimal and vice versa.
- Integrate numerical information in alternative representations to make sense of it (interpret '67% leased — only one tenancy remaining for lease. Building ready October 1992', explaining that it probably means the building has just three tenancies, of which two are leased).

Applying numbers

At level 7, a student:

7.14 Understands the nature of a rate, and chooses, calculates and compares with ratios and rates, including situations involving direct and inverse proportion.

Evident when students, for example:

- Choose ratios, rates or percentages to compare two situations (use price and concentration to compare cordial; compare stocking rates of stations as well as the number of cattle on each).
- Convert from one rate to another to compare and order them, saying which is better, greater, faster (compare consumption of 3 litres per second and 3 kilolitres per hour).
- Use rates to convert from one quantity to another to illuminate comparisons (convert winning margins to distances from times or times from distances to give a feel for the closeness of a race: 0.002 sec. in a cycle race is about the radius of a wheel and two car lengths in a Formula 1 car race is about a tenth of a second).
- Integrate information from two rates (derive the increase in the world's population in one year from birth and death rates).
- Recognise inverse proportion — inverse variation — in familiar situations (realise that the time taken to save an amount is inversely proportional to the amount saved each week).
- Understand inverse proportion as a special case of one variable increasing as the other decreases and identify the assumptions needed to use inverse proportion.
- Set up statements of equality between ratios involving quantities directly or inversely proportional.
- Understand the idea of a variable rate involving changes in a rate over time — inflation rates, growth rates— and use fixed and variable rates to hypothesise about a quantity in the past or future (predict movements in prices of items, assuming a fixed inflation rate of 10% per annum and compare with actual data).

Mental computation | Written computation | Calculators

At level 7, a student:

7.15–7.17 Undertakes efficient computations on positive and negative numbers of any size, including rearranging formulas and quoting results to a suitable level of accuracy.

Evident when students, for example:

- Calculate with positive and negative numbers.
- Consider the precision of the initial information and uses to which results are to be put in choosing levels of accuracy.
- Plan calculations for substituting into formulae, which may be unfamiliar, taking account of sequences of operations and the idiosyncrasies of the particular calculating technology used.
- Rearrange formulae into a form more suitable for repetitive calculation (rearrange $I = Prt/100$ to find t, rewrite $(x + y)/xy$ as $1/x + 1/y$ as the basis for a routine for a scientific calculator).
- Meet specified or chosen levels of accuracy.
- Use technology to undertake computational tasks involving iterative processes, including evaluating a result at several stages (use a spreadsheet to calculate the balance of a loan for 20 successive months).
- Use in-built functions in a calculator to compute basic statistics and trigonometric ratios.
- Take into account truncation errors in various calculators.

Level 8 outcomes:

8.11–8.17 Searches for and uses representations for number and operations that will assist the solution of problems by highlighting patterns in numbers or by reducing complexity and computational load.

LEVEL 7 Measurement

Level 6 outcomes:

6.18 & 6.19 Decides what measurements are needed to complete a practical task and how to obtain them and make or collect measurements to the planned level of accuracy.

6.20 Unprompted, estimates in situations in which it is sensible to do so and judges the reasonableness of estimates.

6.21 Integrates information from several sources to determine time and duration of time and to plan and synchronise events.

6.22a Understands and uses relationships involving length, area and volume of quadrilaterals and circles, prisms and pyramids.

6.22b Understands and uses similarity and Pythagoras's theorem to solve problems involving right triangles and scale drawing.

Choosing units

7.18 No outcome at this level in this sequence.

Measuring

7.19 No outcome at this level in this sequence.

Estimating

At level 7, a student:

7.20 Appreciates that all measurements involve error and estimates the extent of uncertainty in direct and indirect measures.

Evident when students, for example:

- Describe the error range intrinsic to the choice of unit (5 cm long means a centimetre is the unit and the thing is between 4.5 cm and 5.5 cm long, while 5.0 cm long means a millimetre is the unit and the thing is between 4.95 cm and 5.05 cm long).
- Identify possible extrinsic sources of error in their and others' work (recognise that reaction time, sound delay and the error in reading the stopwatch are all factors in timing a race).
- Estimate the effects of different components of error (the contributions to uncertainty of each of the error components in the timing of a race).
- Discuss expected errors, making and justifying decisions about the acceptable level of uncertainty in different contexts (calculate the upper and lower limits expected in a 1 tonne load of sand, as opposed to medicine dosages).
- Describe the compounding effect of errors in calculations involving measurements (calculate the upper and lower limits to consider when ordering concrete for a slab measured as 0.01 m x 4.3 m x 2.1 m).
- Critique conclusions drawn in practical activities from the point of view of inherent errors.

Time

7.21 No outcome at this level in this sequence.

Using relationships

At level 7, a student:

7.22a Selects and uses, both directly and indirectly, formulae for length, area and volume of figures and objects (including for spheres).

Evident when students, for example:

- Use formulae for finding the surface area and volume of spheres.
- Use formulae indirectly (calculate the radius of a cylinder from information about its height and volume).
- Select and sequence the use of formulae to use information from using one formula in another (use the known height and volume of a cylinder to calculate its surface area).
- Describe how one quantity varies with another by inspecting the formula that relates them ($d = 4.9t^2$ tells us that as the time doubles the distance fallen will quadruple, $V = \pi r^2 h$ tells us that the volume is proportional to the height).
- Recognise situations to which a formula is applicable, but not immediately or obviously so (see a baseball diamond as a square and the distance from home to second as the hypotenuse of a right triangle, hence use the Pythagorean result to find the distance from home to second, given distances between bases).
- Produce objects according to sets of measurement specifications where use of formulae is required (make a cylinder with height 15 cm and diameter 6 cm, a cube which will hold 500 mL).

7.22b Understands and uses similarity relationships in and between figures and objects, including with trigonometric ratios.

Evident when students, for example:

- Use similarity to find lengths of sides of polygons.
- Understand that the sin, cos and tan functions in their calculator give ratios of sides in right triangles, and use sine, cosine and tangent to find missing sides or angles in right triangles.
- Use their knowledge of trigonometric ratios in right triangles to solve problems involving triangles that are not right.
- Understand and use the effect of scaling the linear dimensions on the perimeter, area and volumes of shapes to solve problems (use knowledge that if you double the radius of a circle you quadruple its area to compare the cost of pizzas of diameter 20 cm and 40 cm without calculation).
- Apply their understanding of the effect on scaling linear dimensions of an object to explain natural phenomena (why babies dehydrate more quickly and eat more often than adults).
- Investigate and explain how an instrument based on similarity works (thumbstick, clinometer, binoculars, microscope).

Level 8 outcomes:

8.18–8.22 Selects and integrates mathematical ideas, relationships and information, directly and indirectly, to solve practical and analytic measurement problems.

LEVEL 7 Chance and Data

Level 6 outcomes:

6.23 Estimates probabilities and proportions based on primary or secondary data collection and assigns probabilities for one- and two-stage events by reasoning about equally likely events.

6.24 Plans experiments and surveys, collaboratively and independently, deciding how to collect data consistently and the sources and types of data.

6.25 Organises data in diagrams, tables and databases to help answer questions and generate new ones, planning class intervals and fields collaboratively and individually.

6.26 Displays and summarises data to show location and variability (including when some grouping of data is required) to compare data sets and to show relationships in one data set.

6.27 Interprets collected and published data from tables, diagrams, plots, graphs, prose and databases to make comparisons and describe relationships.

Understanding, estimating and measuring chance variation

At level 7, a student:

7.23 Estimates probabilities, proportions, means and medians based on primary and secondary data collection and assigns probabilities using complementarity and independence.

Evident when students, for example:

- Interpret 'and', 'or' and 'not' when used to describe events (explain that the probability of winning first and second prize is less than of winning first prize).
- Use complementary events to assign probabilities (calculate the probability of getting at least one head in five tosses of a coin using 1 – 'the probability of getting no heads in five tosses').
- Distinguish dependent and independent events and relate to everyday interpretations of phenomena.
- Estimate means and medians from simulation data (given data on the take-up rate for offers of places in a computer course, use simulation data to find the mean number of places needed to offer to fill the class).
- Estimate probabilities and proportions from simulation data (estimate the probability that a class would be over-full if a particular number of places were offered).
- Assign conditional probabilities based on data in two-way tables (given survey data giving sex of respondent and whether they voted Yes or No to lowering the driving age, determine the probability that a person voted No, that a female voted No, that a person is male given s/he voted No).
- Investigate and report accurately on the probabilities involved in games of chance, raffles.

Collecting data

At level 7, a student:

7.24 Plans experiments, simulations and surveys, collaboratively and independently, considering the appropriateness and quality of observations and the suitability of samples.

Evident when students, for example:

- Choose and use random number devices such as coins, spinners, tables, telephone books, calculator random numbers.
- Adapt a simple model to simulate a situation involving chance processes (use a set of random numbers to simulate the collection of sets of cards when one card is placed in each chocolate bar; use weather data to design a spinner to investigate run lengths of dry days).
- Identify faults in proposed or conducted questionnaires (leading, ambiguous, misleading, presuming or potentially embarrassing questions, too reliant on memory) and investigate the effect on responses of two forms of a questionnaire prepared from alternative viewpoints (on gun law reform).
- Decide when a sample is adequate and when a census is needed (make a case for using a sample or a census of students' views in the school on matters such as which sports the school should offer, election of representatives on the School Council).
- Select types of samples, such as convenience, random, self-selection, stratified random (describe various ways in which a sample of 50 students in the school could be chosen).

Organising data

At level 7, a student:

7.25 Compares, chooses and uses methods of organisation to suit the type of data and the questions asked.

Evident when students, for example:

- Decide whether and how to group measurements, considering the purpose, the range of measurements and the number of classes (for head sizes for making hats, the group interval should balance how well the hats need to fit with the number of different hat sizes it is economical to make).
- Experiment with different class intervals for histograms of the same set of data and report the effect on interpretations.
- Design nested and layered tables to represent data involving more than two variables (plan how to represent information about age, gender and music preferences in one table).
- Develop a reasonable size data base, identifying variables that will provide information about the questions and hence the fields to be used (to plan radio programs, set up a data base, continuously updated, to use for the top 40 in the pop charts).
- Plan and use a spreadsheet to organise data (about the amount of time females and males spend at the computer keyboard and the extent of their interactions).

Displaying and summarising data

At level 7, a student:

7.26 Displays and summarises data to show location, variability and association, and links displayed data with measures of location, variability and association.

Evident when students, for example:

- Use interquartile box plots to compare the location and variability of several sets of univariate data (the scoring patterns of netball players in a team for overall scores, consistency).
- Compare the informativeness of different graphs of the same data (compare the representation of the proportion of Aboriginal and non-Aboriginal Australians in different age groups in a table, double-column graphs, back-to-back stem plots, box plots).
- Match 'by eye' various line plots with provided means and standard deviations, and scatter plots with provided correlation coefficients.
- Describe scatter plots as suggesting positive, negative or no association, and place informal expressions of correlation on a scale from –1 to 1 (perfect negative correlation, strong negative correlation, weak negative correlation, no correlation).
- Use informal methods to improve lines of 'good fit' (have the line pass through the mean of each variable after removing outliers).
- Describe the effect of outliers on summary statistics (use a scientific calculator to investigate measures of location, variability and association for data sets which they vary systematically).
- Use either mean or median smoothing to smooth a plot of data over time (when studying temperature variations).

Interpreting data

At level 7, a student:

7.27 Selects and interprets information from collected and published data to construct arguments.

Evident when students, for example:

- Describe the assumptions made in the collection of experimental and simulation data.
- Comment upon potential sources of bias, such as samples, question bias, measurement errors, recorder bias (in newspaper polls of various kinds, including phone-ins).
- Interpret information provided in complex nested and layered tables, such as those provided by the ABS.
- Interpret box plots, focusing firstly upon where the boxes are located and their length and then on the details.
- Interpret scatter plots, considering whether there is positive, negative or no association, and noting clusters of points or points that do not appear to follow the general pattern.
- Select aspects of their data to highlight in constructing arguments to support their conclusions.
- Demonstrate how data can be manipulated to different ends (role-play opposing positions on a public issue such as immigration, using the same data set to develop arguments).
- Evaluate arguments of others, commenting on the connections between data and the conclusions reached.

Level 8 outcomes:

8.23–8.27 Comments critically on the strengths and weaknesses of various forms of data collection, analysis and display in terms of what information can be obtained from them and what conclusions might be drawn.

LEVEL 7 Algebra

Level 6 outcomes:

6.28 Uses basic conventions of algebraic notation in representing situations involving a variable quantity and explains why two linear expressions are equivalent.

6.29a Plots, sketches and interprets graphs, considering points, interval lengths, increases and decreases over an interval, and slope.

6.29b Recognises and represents at least linear and square relationships in tables, symbols and graphs and informally describes how one quantity varies with the other.

6.30 Sets up equations to represent one constraint in a situation, solves equations of the form $ax \pm b = cx \pm d$ and $ax^2 \pm b = c$ using guess, check and improve and graphical methods, and solves linear equations using analytic methods.

Expressing generality

At level 7, a student:

7.28 Uses algebraic notation in representing general properties of numbers and relationships between variables and establishes equivalence, including by using the distributive property and inverses of addition and multiplication.

Evident when students, for example:

- Use the distributive property and inverses of addition and multiplication to rewrite complex linear expressions in more useful forms (write $10n + (26n + 12) + 0.25[10n + (26n + 12)] + 20$ for production costs for an item, and simplify to facilitate computation of costs for specific values of n).
- Identify a variable in a situation and represent it symbolically, including where products, opposites and reciprocals of variables are needed (express the area of a rectangle as $m(m - 3)$ or a diving formula as $T = {}^{120V}/_{d}$).
- Conjecture about whether symbolically expressed rules are equivalent by substitution of values and test on a graphics calculator (graph $x^2 + 4x + 4$; $x(x + 4) + 4$; $x(x + 2) + 2(x + 2)$).
- Use the distributive property and inverses of addition and multiplication to decide whether symbolically expressed rules involving products, opposites and reciprocals of variables are equivalent.
- Show their understanding of identities such as $(a + b)^2 = a^2 + 2ab + b^2$ by referring to number patterns and spatial arrangements (dissect and rearrange the pieces of a square of side length $a + b$).
- Recognise and use special cases of $(a \pm b)^2 = a^2 \pm 2ab + b^2$ and $a^2 - b^2 = (a - b)(a + b)$ (based on the identity, see that $a^2 - 1 = (a - 1)(a + 1)$ and $4a^2 - 9b^2 = (2a - 3b)(2a + 3b)$; justify conclusions in investigating patterns in: $6^2 - 5^2 = \ldots$; $56^2 - 45^2 = \ldots$; $556^2 - 445^2$).
- Use linear, quadratic and reciprocal expressions in making general arguments (write any two odd numbers as $2n + 1$ and $2k + 1$ (n, k whole) and show that the product of two odd numbers is odd).

Function

At level 7, a student:

7.29a Plots, sketches and interprets graphs in four quadrants, considering local and global features, including maxima and minima and cyclical changes.

Evident when students, for example:

- Sketch graphs involving negative values both from data collection and by visualising familiar situations (model the height of a sewing machine needle over time with the central position as zero).
- Consider the range of values to be fitted and the use to which a graph is to be put when selecting scales for axes (a scale sufficient to get a feeling for how height changes from birth to adulthood may be inadequate for comparing average heights of boys and girls or if particular values are to be read from the graph).
- Use interpolation, extrapolation, slope, maxima and minima in interpreting graphs, relating local and global features to the original situation (describe what a graph of vertical height against flight time shows about a plane journey, comparing graphs of journeys from Canberra to Sydney and Darwin to Sydney).
- Redraw graphs to reflect changes in conditions (to show an hour spent in a holding pattern over Sydney at the end of a flight from Melbourne).
- Draw lines and curves of 'good fit' by eye and use informal methods for improving the fit such as having the curve pass through the mean of each variable after removing obvious outliers (to relate repair costs to vehicle age).

7.29b Recognises and represents at least linear, reciprocal, exponential and quadratic functions in tables, symbols and graphs and describes assumptions needed to use these functions as models.

Evident when students, for example:

- Recognise from a graph that a relationship is, exactly or almost, linear, reciprocal, square or exponential (from several formulae select those which match particular graphs).
- Understand how the ideas of inversely proportional, inverse variation and reciprocal function are linked.
- Recognise from the algebraic form that a relationship is linear, reciprocal, square or exponential and use this to help sketch the function.
- Write a symbolic expression for a line fitted to data (express as $y = mx + c$ or $ax + by = c$ by measuring rise, run and y intercepts, or by using two points and solving simultaneous equations).
- Look for difference and ratio patterns in tables of data to decide whether a sequence could be linear, square, cubic or exponential.

Equations and inequalities

Level 8 outcomes:

8.28–8.30 Readily identifies algebraic form or structure in mathematical situations, recognising particular situations as instances of more general ones and moving readily between the general case and specific instances.

- Express a recursively described rule in general terms (express the rule 'Start with $100 and double each year' as $p = 100 \times 2^{n-1}$).
- Describe the assumptions needed to use a formula or method (in using the formula $I = Prt/100$, it is assumed that the interest rate will remain constant over the period of the loan).
- Use symbolic expressions to represent and explain growth rates in physical situations (comparing linear growth of radius (r, $2r$, $3r$) with non-linear growth of area (πr^2, $4\pi r^2$, $9\pi r^2$)).

At level 7, a student:

7.30a Sets up equations and pairs of simultaneous equations to represent constraints in a situation and solves, using guess, check and improve, graphical and, for linear equations, analytic methods.

Evident when students, for example:

- Formulate equations in one or two variables with linear, quadratic, reciprocal and exponential expressions, that is, write equations for 'One number is seven less than a second number, double the former number is also triple the latter' or 'After a certain number of years, the population had halved'.
- Use graphical and, where practical, guess, check and improve methods for finding approximate real solutions to equations and pairs of simultaneous equations involving linear, quadratic, reciprocal and exponential expressions.
- Choose a level of accuracy for a solution suited to the context and use a calculator to refine solutions found graphically.
- Use analytic methods to solve linear equations and simple cases of simultaneous linear equations.
- Use the context to choose and justify which of possible solutions to an equation are relevant (solving $f(x) = 0$ may produce two possible solutions, one or neither of which is meaningful).
- Use an analytic method to solve quadratic equations from among completing the square, quadratic formula or factorising trinomials.
- Decide whether quadratic expressions are always, sometimes or never equal ($x^2 + 4x$ and $x(x + 4)$; $a^2 + b^2$ and $(a + b)^2$.

7.30b Sets up inequalities to represent one or two constraints in a situation and generates a complete set of numbers or number pairs that satisfy the constraints.

Evident when students, for example:

- Describe numbers or pairs of numbers or shade regions of a plane to show all values that satisfy verbal constraints (a room must be between 9 m^2 and 16 m^2 and width no more than 4 m).
- Formulate inequalities (linear, simple powers and exponential) to represent constraints (three punters lose $100 overall and each loses more than $25; the total cost of production must be no more than $1000 but labour is typically double the cost of materials).
- Use substitution to decide the status of particular pairs of values with respect to a symbolically expressed inequality (does (7, 3) satisfy $a + b \leq 10$?).
- Describe a situation that could have led to a particular inequality (make up a story that fits the inequality $5b < 20$ or $x^2 > 25$).
- Investigate and report on the conditions under which statements are true ($4 + x = 10$, $a - 4 < 3$, $x^2 > 1$, $|-7| < 5$, $3x < 4x$).
- Solve linear inequalities in one variable analytically, (backtrack to solve $5n + 3 < 33$).
- Use graphs to solve one or two inequalities (shade the plane to show $y = 2x$, $y < 2^x$ and $y > 2^x$, respectively; shade the plane to show where $y < 2x + 3$ and $y < 4 - x$, simultaneously).

LEVEL EIGHT

LEVEL 8 Statement

Students at level eight use the same content as students at level seven but do more with it. They have some understanding of the systematic nature of a geometry, draw flexibly on the results about shapes, transformations and locations to solve problems, and are more likely than students at level seven to identify key spatial features of a geometric problem essential to its solution.

At this level, they look for 'the best' representations for number and operations and choose forms of numbers and operations that highlight the pattern they are seeking.

Students select and integrate mathematical ideas, relationships and information from diverse sources to solve measurement problems. They realise that the best test for correctness of a solution to a measurement problem is likely to be to try it out in practice. They also appreciate that they need to be able to argue in general why a solution works.

At this level, students are able to comment critically on the strengths and weaknesses of data collection, analysis and display and discuss possible conclusions.

Students readily identify algebraic form or structure in mathematical situations and move readily between the general case and specific instances. They also choose to express variables algebraically to explain general relationships.

Students show persistence, autonomy and flexibility in mathematics. They are able to sustain themselves over several sessions and take responsibility for their own work. They readily draw on their repertoire of mathematical skills, deciding which tools to apply and why. They question the assumptions in a mathematical model and ask whether it is suitable in a given situation. Having solved a particular problem, they are likely to ask themselves whether the same strategy can be used elsewhere. They have some appreciation of the importance of convincing themselves and others of the truth of mathematical propositions. They understand the distinction between arguments based on deduction and induction, and have some facility with deducing new mathematical results from given information. They make fluent use of mathematical notation in solving problems and in presenting mathematical arguments.

LEVEL 8
Table of outcomes

8.1 Shows persistence, autonomy, flexibility and self-reliance when working mathematically.	8.2 Produces mathematical arguments to convince others of the truth of propositions, including those involving deductions from known information.	8.3 Uses problem-solving strategies that include those based on generalising from one problem situation to another and rethinking problem conditions and constraints.	8.4 Formulates models by making useful and simple assumptions, collecting data needed and representing the relevant relationships in mathematical terms.	8.5 Makes fluent use of mathematical notation in solving problems and presenting arguments succinctly, coherently and in conventional forms.	8.6 Appreciates that there is a relationship between mathematics and social conditions and values, commenting on the role of mathematics in describing and shaping aspects of our lives.
See page 132	See page 132	See page 132	See page 133	See page 133	See page 133

8.7–8.10 Recognises the systematic nature of a geometry and draws flexibly upon, and sees connections between, results about shapes, transformations and locations.

See page 134

8.11–8.17 Searches for and uses representations for number and operations that will assist the solution of problems by highlighting patterns in numbers or by reducing complexity and computational load.

See page 135

8.18–8.22 Selects and integrates mathematical ideas, relationships and information, directly and indirectly, to solve practical and analytic measurement problems.

See page 136

8.23–8.27 Comments critically on the strengths and weaknesses of various forms of data collection, analysis and display in terms of what information can be obtained from them and what conclusions might be drawn.

See page 137

8.28–8.30 Readily identifies algebraic form or structure in mathematical situations, recognising particular situations as instances of more general ones and moving readily between the general case and specific instances.

See page 138

LEVEL 8 Working Mathematically

Level 7 outcomes:

7.1 Poses, clarifies and refines mathematical questions to help understand or guide the investigation of a situation.

7.2 Makes generalisations by abstracting common mathematical features from situations, tests with additional cases and explains why generalisations must be true.

7.3 Uses problem-solving strategies that include identifying and working on related problems or sub-problems.

7.4 Applies standard methods or models, comparing and choosing between alternatives, including by considering assumptions needed and results obtained.

7.5 Uses conventional mathematical language to help give clear and logical accounts of mathematical work.

7.6 Makes links between the development and use of mathematical ideas and the conditions and concerns of the individuals and communities that produce them.

Investigating

At level 8, a student:

8.1 Shows persistence, autonomy, flexibility and self-reliance when working mathematically.

Evident when students, for example:

- Refocus or restate the constraints in a situation to clarify the nature of the problem.
- Respond positively to confusion as evidence that they do not yet have a complete understanding of a particular mathematical idea or aspect of a task.
- Use ambiguity productively to help refine their thinking (use a suggested definition of a quadrilateral as 'a figure formed by joining four line segments together end to end to form a closed shape' to begin an investigation of the properties and defining characteristics of quadrilaterals).
- Try alternative lines of investigation when initial attempts are unproductive.
- Ask whether their solution is the only possible one and attempt to find other solutions— 'Do I have them all?' and 'How can I tell?' (investigate similar shapes constructed on the sides of a right triangle and conjecture that Pythagoras' theorem can be generalised).
- Ask themselves questions such as 'How confident am I about the solution?', 'What would a reasonable answer be?', 'Is each step of the solution correct?' and 'Is there another method I could use to check my answer?'

Conjecturing

At level 8, a student:

8.2 Produces mathematical arguments to convince others of the truth of propositions, including those involving deductions from known information.

Evident when students, for example:

- Search for an explanation of why a pattern observed in data 'must' hold (find the number of diagonals for a quadrilateral, pentagon, hexagon and heptagon), use difference patterns to help find a relationship to fit the data, test for other cases, then search for a general argument and justify the rule.
- Check a range of different cases to find a counter-example (testing a generalisation about the diagonals in polygons by testing with concave polygons and for the case of a triangle, which has no diagonals).
- Distinguish a general argument from one based on specific cases and distinguish the colloquial use of the word 'generalisation' as meaning 'usually true' from the mathematical use of the word as meaning 'always true', or true within a specified domain.
- Identify errors in logic in their own work and that of others (explain what is wrong with the following: $x(x - 2) = 0$, so $x = 0$).
- Generate results deductively from information provided about a situation (given a diagram with some information on it, deduce other information about the shape).

Using problem-solving strategies

At level 8, a student:

8.3 Uses problem-solving strategies that include those based on generalising from one problem situation to another and rethinking problem conditions and constraints.

Evident when students, for example:

- Recognise a problem as analogous to another problem and therefore likely to be amenable to a similar strategy (having solved a pegboard problem, try a similar strategy on the Tower of Hanoi or a problem involving moving people from one side of a river or another).
- Rethink problem conditions by changing the point of view (to allocate 10 people to 12 positions according to specified criteria, change the focus from the 10 positions to be filled to the 2 positions to remain unfilled).
- Present the problem in a different way, using strategies such as changing the form of representation ('Perhaps I should think about it diagrammatically or geometrically') or the notation ('Would it be better if I expressed it in common fractions?').
- Check for hidden assumptions in their own reasoning (that streets cross at right angles) and break set by discarding those assumptions.
- Distinguish essential constraints and desirable features in problem situations.
- Explain their strategy for a problem in general terms to enable it to be applied to new tasks (for a problem such as 'Is 10% discount (for employees) followed by 15% off (for the sale) better or worse than 15% off then 10% off', solve the problem as posed but then look for the general principle involved).

Applying and verifying

At level 8, a student:

8.4 Formulates models by making useful and simple assumptions, collecting data needed and representing the relevant relationships in mathematical terms.

Evident when students, for example:

- Distinguish significant from insignificant factors in a problem and identify the relevant variables (when dividing by 12 to convert from annual to monthly interest rates, recognise the importance of the compounding effect for borrowers and the insignificance of the differing lengths of months; in investigating traffic flow, list variables such as the change time for traffic lights).
- Make assumptions about the type of variation in a situation (assume that heat loss is proportional to skin area, that tea cools exponentially, that suitcases will be rectangular prisms).
- Investigate approximate formulae and rules of thumb so as to suggest better models (given that the effective interest rate is approximately twice the flat rate, use a spreadsheet to test the formula for various periods and interest rates and try alternatives such as E% = 2.1 x F%).
- Understand that alternative assumptions will result in altered predictions (in extrapolating from a downward trend in record times for a sporting event, discuss the effects on the predictions made if the general use of bodybuilding steroids were allowed).
- Use the context to choose which of the possible solutions to an equation are relevant explaining the reason (solving f(x) = 0 for x may produce two possible solutions, only one of which is meaningful).

Using mathematical language

At level 8, a student:

8.5 Makes fluent use of mathematical notation in solving problems and presenting arguments succinctly, coherently and in conventional forms.

Evident when students, for example:

- Appreciate the power of mathematical symbolisation for expressing ideas and show a readiness to represent ideas symbolically and work with abstractions.
- Work fluently with algebraic forms of representation, readily manipulating and rearranging expressions to achieve a purpose.
- Provide literate accounts of their own mathematical work— that is, accounts which are logical, make appropriate use of terms and notations, and are grammatically correct.
- Strive for clarity, simplicity and economy in their presentation of solutions to problems and proofs.
- Read to find the meaning of unfamiliar notations or expressions in mathematics (they may know of the rules for exponents but not have heard them described as index laws, or they may be unfamiliar with some notations for functions come across in a resource).
- Choose an appropriate means for communicating their mathematical work, taking into account the audience (peers, the rest of the school) and purpose (to teach some mathematics, to report results, to convince others, to persuade).

Working in context

At level 8, a student:

8.6 Appreciates that there is a relationship between mathematics and social conditions and values, commenting on the role of mathematics in describing and shaping aspects of our lives.

Evident when students, for example:

- Make links between cultural mores and social conditions and some of the mathematics developed and used within the community (draw on information about the Balinese religion and the rice-based economy to hypothesise about the 'obsession' of Balinese people with numbers; compare the approach to navigation by Europeans and Pacific Islanders and offer reasons why in only one case the need to navigate led to certain mathematical developments).
- Identify instances of mathematical models that have come to assume the status of 'real-world' phenomena (IQ, inflation, tertiary entrance scores, batting averages), describing what reality these models are intended to represent and how the model itself may form our views of the world.
- Compare the mathematical features of different ways of representing aspects of the social world (family relationships for Australians of European descent are typically shown on a tree, a family line, usually through the male line only, with branches, while for Australians of Aboriginal descent they are typically shown with cyclic patterns).
- Describe how mathematics can help develop, convey and assess ideas and arguments about social and physical phenomena and can therefore shape aspects of our thinking.

LEVEL 8 Space

Level 7 outcomes:

7.7 Draws on properties of shapes and transformations to plan how to meet specifications requiring the accurate construction or placement of figures and objects.

7.8 Visualises, constructs and describes paths and regions using conventional geometric language, including that based on coordinates.

7.9 Investigates and uses relationships in and between classes of figures, including quadrilaterals and circles.

7.10 Analyses translations, reflections, rotations and enlargements and relates their properties to similarity and congruence.

Outcomes 7–10 cover all organisers of this strand at this level.

At level 8, a student:

8.7 – 8.10 Recognises the systematic nature of a geometry and draws flexibly upon, and sees connections between, results about shapes, transformations and locations.

Evident when students, for example:

- Produce chains of arguments to prove statements based on a given set of geometric axioms or assumptions (traditional or transformational axioms for Euclidean geometry, a finite geometry).
- Recognise the same spatial idea in different forms and appreciate the flexibility and power this brings to the solution of problems (how the ideas of parallelism and congruence are represented in coordinate, traditional Euclidean and transformational approaches, respectively).
- Readily identify key spatial features of a geometric problem essential to its solution, including disembedding simple shapes from within complex ones (identify the triangles within a diagram that are the key to proving two segments congruent).
- Rearrange the components in spatial problems to try to turn a problem into something more familiar (note that the problem involves a right triangle and area, think about Pythagoras and try to recast the problem into one involving this relationship).
- Draw on ideas from various parts of mathematics to apply to spatial problems.
- Choose between different forms of representation of a spatial problem to prove or disprove generalisations (select the most helpful approach to establishing that the altitudes of a triangle are concurrent).

LEVEL 8 Number

Outcomes 11–17 cover all organisers of this strand at this level.

At level 8, a student:

8.11 – 8.17 Searches for and uses representations for number and operations that will assist the solution of problems by highlighting patterns in numbers or by reducing complexity and computational load.

Evident when students, for example:

- Appreciate the distinction between exact and approximate solutions (recognise 1/3 as exactly equal to $0.\dot{3}$,) and more precise than 0.333333 and π as more exact than any common or decimal approximation to it).
- Realise that when searching for a pattern analytically, exact forms of numbers are likely to prove more illuminating than approximations (in calculating special cases, leave solutions in the form √n rather than finding decimal approximations; when dealing with the effect of adding 10% each year, consider whether thinking of it as finding 110% or x 1.1 is more illuminating).
- Search for and use the 'easy' way to carry out calculations (recognise 39 x 41 as a difference of squares and use for mental computation; search for a way to reduce the complexity of input when developing a spreadsheet formula).
- Identify the more general problem in a special case of an arithmetic problem (asked is it better to add 10% tax and then take 15% discount, solve the specific problem, say, 'It made no difference, but is it always so?', and express the computation as a series of multiplications to find out).

Level 7 outcomes:

7.11 Selects a suitable form of number representation, explains choice and moves freely between representations.

7.12 (See outcome 7.28)

7.13 (See outcome 7.30)

7.14 Understands the nature of a rate, and chooses, calculates and compares with ratios and rates, including situations involving direct and indirect proportion.

7.15–7.17 Undertakes efficient computations on positive and negative numbers of any size, including rearranging formulas and quoting results to a suitable level of accuracy.

LEVEL 8 Measurement

Level 7 outcomes:

7.18 No outcome at this level

7.19 No outcome at this level

7.20 Appreciates that all measurements involve error and estimates the extent of uncertainty in direct and indirect measures.

7.21 No outcome at this level

7.22a Selects and uses, both directly and indirectly, formulae for length, area and volume of figures and objects (including for spheres).

7.22b Understands and uses similarity relationships in and between figures and objects, including with trigonometric ratios.

Outcomes 18–22 cover all organisers of this strand at this level.

At level 8, a student:

8.18 – 8.22 Selects and integrates mathematical ideas, relationships and information, directly and indirectly, to solve practical and analytic measurement problems.

Evident when students, for example:

- Develop their own methods for estimating areas and volumes of irregular or complex shapes, including by decomposing the shape into shapes for which they can find the area and volume.
- Select and use a range of spatial relationships and measurement formulae to determine exactly and to justify the measurements needed to meet design specifications (make a set of three of the Platonic solids, all with the same volume; construct a cone 10 cm high that fits exactly inside a sphere of 15 cm diameter).
- See the need to justify their solution to measurement problems (draw on geometry to justify that the diagonal of the square inscribed in a circle is a diameter).
- Generalise from their knowledge of measurement relationships to conjecture about other related shapes (knowing that all parallelograms of a particular base and height have the same area as the rectangle on that base and height, conjecture that slant prisms of the same height and on the same base will have the same volume).
- Use relationships between similar shapes indirectly to produce objects according to constraints (select three containers and make scale models of them with half the capacity).
- Appreciate the significance of the accuracy of initial measurements for the level of accuracy of derived measurements (understand that in general answers will be no more accurate than the initial measurements, but also see that this need not always be so —consider division).

LEVEL 8 Chance and Data

Outcomes 23–27 cover all organisers of this strand at this level.

At level 8, a student:

8.23 – 8.27 Comments critically on the strengths and weaknesses of various forms of data collection, analysis and display in terms of what information can be obtained from them and what conclusions might be drawn.

Evident when students, for example:

- Distinguish plausible, possible and impossible interpretations of data, including when critiquing the claims of others, as in media articles, and when dealing with emotive issues.
- Distinguish without prompting between conclusions that reflect personal opinion and those supported by particular data (will say, 'I think that conclusion will turn out to be right but we don't have the data to support it here', or 'This data supports that conclusion, even though I find it surprising').
- Explain clearly the difference between association and cause-and-effect and apply this consistently when making their own interpretations of data.
- Compare and contrast the information provided by 50% and 90% box plots.
- Present a convincing argument for the advantages and disadvantages of certain types of summary statistics and displays for representing particular data.
- Understand the significance of, and take account of, the definitions of terms used for the conclusions reached (take into account different ways in which an expression such as 'out of work' is operationalised for the purpose of investigating part-time work among school students).
- Understand that when samples are stratified, the basis for the stratification can determine the conclusions reached, and take this into account in their interpretations of data (in surveying the school, a stratified sample based on gender, age group, boarder or non-boarder, and course enrolments could produce different results).
- Demonstrate the effect of different assumptions on conclusions reached by setting up two different simulations to model a situation and comparing the results.
- Recognise the element of chance in data collection caused through natural variability in populations and measurement errors.

Level 7 outcomes:

7.23 Estimates probabilities, proportions, means and medians based on primary and secondary data collection and assigns probabilities using complementarity and independence.

7.24 Plans experiments, simulations and surveys, collaboratively and independently, considering the appropriateness and quality of observations and the suitability of samples.

7.25 Compares, chooses and uses methods of organisation to suit the type of data and the questions asked.

7.26 Displays and summarises data to show location, variability and association, and links displayed data with measures of location, variability and association.

7.27 Selects and interprets information from collected and published data to construct arguments.

LEVEL 8 Algebra

Level 7 outcomes:

7.28 Uses algebraic notation in representing general properties of numbers and relationships between variables and establishes equivalence, including by using the distributive property and inverses of addition and multiplication.

7.29a Plots, sketches and interprets graphs in four quadrants, considering local and global features, including maxima and minima and cyclical changes.

7.29b Recognises and represents at least linear, reciprocal, exponential and quadratic functions in tables, symbols and graphs and describes assumptions needed to use these functions as models.

7.30a Sets up equations and pairs of simultaneous equations to represent constraints in a situation and solves, using guess, check and improve, graphical and, for linear equations, analytic methods.

7.30b Sets up inequalities to represent one or two constraints in a situation and generates a complete set of numbers or number pairs that satisfy the constraints.

Outcomes 28–30 cover all organisers of this strand at this level.

At level 8, a student:

8.28 – 8.30 Readily identifies algebraic form or structure in mathematical situations, recognising particular situations as instances of more general ones and moving readily between the general case and specific instances.

Evident when students, for example:

- Work flexibly and fluently with algebraic expressions within their experience to rearrange them into more useful forms for a purpose (express $x^2 - 2x + 7$ as $(x - 1)^2 + 6$ to facilitate curve sketching).
- Recognise a familiar algebraic form within an unfamiliar or complex algebraic expression and use this to simplify the expression and draw conclusions (recognise $9x^4 + 12x^2y^2 + 4y^4$ as a perfect square and hence factorise, commenting that, since x is an integer, all the solutions to the problem will be square numbers).
- Choose to express variables algebraically to illuminate or explain general relationships (asked to compare adding 20% sales tax and then taking off 10% discount with taking off 10% discount and adding 20% sales tax, move rapidly from an inspection of one or two special cases to the general case).
- Generalise a learned algebraic technique or idea to related situations (generalise their method of solution for solving two equations in two unknowns to solve a problem involving three equations in three unknowns).
- Identify the main features of a function from its algebraic expression (know that $f(x) = 50 \times 2^x$ will have positive values, be increasing at an increasing rate, have small values for negative x and rapidly increasing values for positive x).
- Recognise the same key idea underlying apparently different mathematics (compound interest, geometric progressions and exponential functions).
- Show a propensity to see particular cases as instances of more general ones and to use this to effect (having learned about families of linear, quadratic, reciprocal and exponential functions and studied the effect of changes in the parameters on their graphs, identify general ideas about how functions are transformed, and predict the effect of changing parameters on other functions not studied in depth).

Appendix

National collaboration in curriculum

National collaboration on curriculum began in June 1986 when the Australian Education Council resolved to support the concept of a national collaborative effort in curriculum development in Australia to make the best use of scarce curriculum resources and to minimise unnecessary differences in curriculum between States.

By 1987, the AEC had identified five priority areas for collaboration: science, numeracy, literacy, languages other than English (LOTE), and English as a second language (ESL).

Three initiatives

The AEC took three significant initiatives in 1988. First, it set up a working party to develop a discussion paper for a national approach to monitoring student achievement 'which takes cognisance of the programs already in place or under development at the State and Territory level'.

Second, it decided to develop a statement of the national goals and purposes of education in Australia.

Third, it set up a project to map the mathematics and general curriculum in all States and Territories through a study of their curriculum documents. The project was managed by the Directors of Curriculum, comprising the senior officers responsible for curriculum in the States and Territories and senior officers in the Commonwealth and the non-government systems. These maps were completed in early 1989.

Landmark decision

In April 1989, the AEC saw the second initiative come to fruition when it endorsed the *Common and Agreed National Goals for Schooling in Australia.*

The AEC also agreed to 'strengthen further the effective collaboration which has occurred to date to enable greater effectiveness and efficiency in curriculum through the sharing of knowledge and scarce curriculum development resources across systems' and to 'remove unnecessary differences in curriculum between systems'.

Building on the work of the third initiative, the AEC decided to develop a statement on mathematics. This would include, within the framework of the agreed national goals, 'the knowledge and skills to which all students are entitled' and the 'agreed areas of strength in curriculum development which might be shared and built upon'. It also decided to set up three more mapping projects, this time in technology, science and English literacy.

Widening scope of activity

In October 1989, the AEC widened the scope of national collaborative curriculum activities to include a mapping of the social sciences and an audit of environmental education materials, followed by a map of the environment as a cross-curriculum study. In May 1990, it decided to include, as one of the terms of reference for the writing of curriculum statements, the principles and objectives of the *National Policy for the Education of Girls.* It also asked for an audit of Aboriginal education curriculum materials. This was followed in December 1990 by approval for statements to be developed in English, technology and science.

Profiles

In mid-1990, the working party set up in 1988 under the first AEC initiative presented its report. It recommended the development of profiles to describe students' learning outcomes at a number of levels. In December 1990, the AEC endorsed the development of two profiles — English and mathematics — by the Australasian Cooperative Assessment Program (ACAP). The profiles would 'provide a framework which can be used by teachers in classrooms to chart the progress of their students, by schools to report to their communities and by systems' reporting on student performance as well as being amenable to reporting student achievement at the national level'.

Eight areas of the curriculum

In April 1991, the AEC launched the projects in their final form by deciding that statements and profiles would be developed for eight broad learning areas, forming a template of the knowledge and processes to be taught and learnt in Australian schools. Most States and Territories had already adopted their own sets of key learning areas, which generally clustered around the eight areas of learning adopted by the AEC.

Project management

Until August 1991, development of statements was being managed by the Directors of Curriculum and the profiles by ACAP. This structure did not provide a close nexus between statements and profiles and so was replaced by the AEC Curriculum and Assessment Committee (CURASS), which managed all projects to completion.

CURASS assumed two major responsibilities: for consultation and for the progressive approval, through consensus, of draft statements and profiles. States, Territories and the Commonwealth had up to two representatives each on CURASS. Non-government systems and sectors were also represented, as were the government and non-government teachers' and parents' organisations. In June 1992, a secretariat was established to support CURASS.

The committee developed a series of guideline papers setting out its position on issues important to the projects. The papers described the nature of statements and profiles, dealt with the roles and functions of the committee itself, set out its processes of consultation, dealt with issues relating to inclusivity, and explored pedagogical implications.

Career education in Australian schools

In 1989, the AEC established a working party on career education, which prepared a document entitled *Career Education in Australian Schools: National Goals, Student, School and System Outcomes and Evaluative Arrangements*. This was referred to CURASS by the AEC in June 1992 'to inform its consideration of career education components within the national collaborative curriculum and assessment framework'.

CURASS decided that career education in general and the document *Career Education in Australian Schools* in particular should be 'taken into consideration in the development of statements and profiles' especially in health and physical education and studies of society and environment.

Inclusivity

In 1992, CURASS decided to undertake two supplementary projects — one for students of English as a second language and the other for students with disabilities. These projects developed the national ESL Scales and the Towards Level 1 section in the profiles and helped ensure that these students had access to the profiles.

In addition, the Commonwealth funded two initiatives aimed in part to achieve high levels of inclusivity in national collaborative curriculum activities.

The first of these was the National Aboriginal and Torres Strait Islander Studies Project, made up of five sub-projects. The first led to the development of the *National Philosophy and Guidelines for Aboriginal Studies and Torres Strait Islander Studies, K–12.* The second secured the inclusion of Aboriginal studies and Torres Strait Islander studies and perspectives within the national statements and profiles. The other three sub-projects aimed to assist teachers in incorporating Aboriginal studies and Torres Strait Islander studies in their programs.

The second Commonwealth-funded project was the Gender Equity and Curriculum Reform Project. This had as one of its components the appointment of a gender equity consultant to each of the national collaborative curriculum projects to ensure that the principles and objectives of the *National Policy for the Education of Girls in Australian Schools* were incorporated into the design briefs, statements and profiles.

The statements and profiles were completed in their present forms in 1993. In July 1993, the AEC referred the documents to States and Territories.

Curriculum CORPORATION

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